Sampling Winners in Ranked Choice Voting

Matthew Iceland[1], **Anson Kahng**[1] and **Joseph Saber**[1]
[1]University of Rochester
miceland@u.rochester.edu, anson.kahng@rochester.edu, jsaber@ur.rochester.edu

Abstract

Ranked choice voting (RCV) is a voting rule that iteratively eliminates least-popular candidates until there is a single winner with a majority of all remaining votes. In this work, we explore three central questions about predicting the outcome of RCV on an election given a uniform sample of votes. First, in theory, how poorly can RCV sampling predict RCV outcomes? Second, can we use insights from the recently-proposed map of elections to better predict RCV outcomes? Third, is RCV the best rule to use on a sample to predict the outcome of RCV in real-world elections? We find that although RCV can do quite poorly in the worst case and it may be better to use other rules to predict RCV winners on synthetic data from the map of elections, RCV generally predicts itself well on real-world data, further contributing to its appeal as a theoretically-flawed but practicable voting process. We further supplement our work by exploring the effect of margin of victory (MoV) on sampling accuracy.

1 Introduction

Democratic systems elicit and aggregate opinions from citizens in order to make collective decisions. The most common way in which they do this is through voting, in which citizens provide structured feedback via ballots, which are then aggregated via a social choice function, also called a voting rule, in order to determine a winner.

One such rule is ranked choice voting (RCV), also known as instant-runoff voting (IRV), single transferable vote (STV), or preferential voting [Spencer *et al.*, 2015]. RCV, or variants thereof, is used in political elections around the world, including Australia, Ireland, New Zealand, and the United States, for a mixture of federal, parliamentary, and local elections. In the United States in particular, RCV is championed by activist groups like FairVote to replace the use of first-past-the-post voting systems and is currently used by 11 million residents: Alaska and Maine use RCV for federal and/or local elections, and an additional 53 cities use RCV for local elections, including New York City's Democractic primary for the mayoral election in 2021 [Horton and Thomas, 2023].

Despite significant activist support for RCV and increasing adoption worldwide, RCV is known to have significant theoretical flaws, notably for being susceptible to monotonicity paradoxes [Felsenthal and Tideman, 2013]. However, in practice, real-world elections do not often resemble worst-case constructions or even synthetic elections generated from statistical cultures [Boehmer and Schaar, 2023], and RCV generally performs well [Graham-Squire and McCune, 2023].

One major concern expressed by activists pushing for more widespread adoption of RCV is that of predicting the outcomes of RCV elections from sampled votes, especially because RCV outcomes can change so drastically as new votes are counted. Our goal is to study how to predict outcomes in RCV elections from samples.

In this work, we aim to explore three central questions about predicting RCV outcomes in sampled elections. First, in the worst case over voting profiles, how poorly can RCV on a sample predict the outcome of RCV on the entire election? Second, can we use insights from elections generated from well-studied statistical cultures to better predict RCV outcomes? And third, how well does RCV predict itself on samples from real-world elections?

1.1 Our Contributions

We begin by examining the worst-case predictive performance of RCV in theory. Surprisingly, we show that, in theory, this performance seems to *decrease* as the sample size grows: For samples of size 1, we obtain a tight bound of probability at least $1/2^{m-1}$ of making the correct decision,[1] but for samples consisting of all but a constant number of votes in the election, we are able to create worst-case instances such that the predictive performance of RCV drops to 0, i.e., using RCV on the sample never yields the same result as evaluating RCV on the entire profile. Additionally, we provide upper bounds on the minimum sample size necessary to guarantee a correct prediction in terms of the margin of victory of the original election, where the margin of victory is defined as the total number of votes that must be changed in order to change the winner of the election.

Next, we examine the performance of a collection of nine voting rules predicting RCV outcomes on synthetic elections generated from a diverse set of statistical cultures from the

[1]Following convention, m is the number of alternatives.

map of elections [Szufa *et al.*, 2020; Boehmer *et al.*, 2021]. We observe that, especially on small sample sizes, RCV is often not the best predictor of itself and that other rules are more reliable.

However, this observation does not hold as strongly for real-world elections. On a range of elections sourced from PrefLib, we find that, on average, RCV is generally the best predictor of itself even on small sample sizes. This does not hold for every individual election, but RCV is even comparable to two ensemble predictors that use the map of elections to boost performance.

1.2 Related Work

The paper most closely related to ours is that of Micha and Shah [2020], which studies the worst- and average-case predictability of social welfare functions (SWFs), which return rankings over candidates instead of winner(s). The authors focus on positional scoring rules (PSRs) and demonstrate that all PSRs except plurality and veto have zero worst-case predictability even with access to a sample of as many as $n-1$ out of n votes. They also include an empirical section that evaluates how well various SWFs can predict each other on two synthetic vote distributions. In our work, we focus on predicting social choice functions (SCFs), in particular RCV; we also consider a significantly more diverse collection of both synthetic and real-world data.

The synthetic data we use is directly inspired (and generated) by the map of elections [Faliszewski *et al.*, 2019; Szufa *et al.*, 2020; Boehmer *et al.*, 2021], which is a principled approach to generating, organizing, and visualizing a diverse set of statistical cultures from which to generate realistic election data.

Another related theoretical paper is that of Bhattacharyya and Dey [2021], where the authors focus on predicting the output of a SCF on an unknown profile through sampling votes. However, the authors assume that votes are sampled with replacement and that there is a margin of victory of at least αn for some constant α. We do not make such assumptions in our worst-case results, and indeed our negative results come in borderline cases. We do consider the margin of victory in RCV elections in our work on bounding the number of samples necessary to make perfect predictions, which draws on work by Cary [2011] and Dey and Narahari [2015].

Further afield, there is also significant theoretical and empirical work on paradoxes in STV [Graham-Squire and McCune, 2023; Tolbert and Kuznetsova, 2021; Donovan *et al.*, 2019], but this work does not focus on sampling.

2 Preliminaries

Let $[n] := \{1, \dots, n\}$. Let $A = \{a_1, \dots, a_m\}$ be a set of m alternatives and $N = [n]$ be a set of n voters. Let $\mathcal{L}(A)$ be the set of all complete and incomplete rankings over A, i.e., (partial) permutations of all alternatives. Each voter $i \in N$ casts a vote $\sigma_i \in \mathcal{L}(A)$. The collection of all n votes is the profile $\vec{\sigma} = (\sigma_1, \dots, \sigma_n)$. We use the notation $a_j \succ_i a_k$ to denote that voter i prefers a_j to a_k and drop the voter subscript when the voter identity is clear.

We focus on *social choice functions (SCFs)* (here interchangeably referred to as *voting rules*), which are functions $f : \mathcal{L}(A)^n \to A$ that, given an input profile, output a winner of the election.[2] Let $s_{g(n)}$ denote a sample taken uniformly at random and without replacement from a complete profile $\vec{\sigma}$ such that $|s_{g(n)}| = g(n)$ for a function $g : \mathbb{N} \to \mathbb{N}$ which, given a number of voters n, returns a sample size in $[n]$. We use $s_{g(n)} \sim \vec{\sigma}$ to denote this process of uniformly selecting a sample $s_{g(n)}$ without replacement from $\vec{\sigma}$. When $g(n)$ is clear, we let $s := s_{g(n)}$ for brevity.

We also define the *worst-case accuracy* of a rule f predicting a rule f' given a sampling function g and a maximum number of alternatives m as

$$\begin{aligned} A_{f,f'}(g, m_0) = \inf p \\ &\text{s.t. } \forall n_0 \in \mathbb{N},\ \exists \vec{\sigma} \text{ with } |\vec{\sigma}| \geq n_0, m = m_0 \\ &\text{s.t. } \Pr_{s_{g(n)} \sim \vec{\sigma}}(f(s) = f'(\vec{\sigma})) \leq p. \end{aligned}$$

Intuitively, this is the minimum probability of f correctly predicting f' on a sample for profiles that consist of exactly m_0 alternatives as n becomes large. When $f = f'$, we let $A_f(g, m_0) := A_{f,f'}(g, m_0)$ for brevity.

2.1 Voting Rules

We define the voting rules in this paper, namely RCV, plurality, Borda, harmonic, Copeland, Minimax, Bucklin, Plurality Veto, and veto, which are discussed in greater detail in [Brandt *et al.*, 2016; Kizilkaya and Kempe, 2022].

RCV proceeds in rounds as follows. In each of $m-1$ rounds, each candidate counts the total number of first-place votes they have, and the candidate with the fewest first-place votes is eliminated.[3] All voters who selected the eliminated candidate as their most-preferred candidate move on to their next most preferred candidate. If, in the course of candidate eliminations, a particular vote has no active candidates remaining, the vote is removed from the election.[4] This process terminates with a single winner. Additionally, it is easy to see that if any candidate has a majority of all first-place votes at any stage of the process, that candidate will win the election.

Plurality, Borda, harmonic, and Veto are all instances of *positional scoring rules (PSRs)*. PSRs are characterized by a scoring vector $\vec{c} = (c_1, \dots, c_m) \in \mathbb{R}^m$, where $c_j \geq c_{j+1}$ for all $j \in \{1, \dots, m-1\}$ and $c_1 > c_m$. Given a profile $\vec{\sigma}$, a PSR with scoring vector $\vec{c}$ assigns a score $sc(a_j) = \sum_{i=1}^{n} c_{\sigma_i(a_j)}$, where $\sigma_i(a_j)$ is the position of a_j in voter i's ranking, σ_i. The alternative with the highest score is the winner.

The scoring vectors of the four rules are as follows: Plurality uses $\vec{c} = (1, 0, \dots, 0)$, Borda uses $\vec{c} = (m-1, m-2, \dots, 0)$, harmonic uses $\vec{c} = (1, 1/2, \dots, 1/m)$, and veto uses $\vec{c} = (0, \dots, 0, -1)$.

The Copeland rule and Minimax both consider pairwise comparisons between alternatives. The Copeland rule chooses the alternative that beats the greatest number of other alternatives in head-to-head comparisons[5], and Mini-

[2]Although SCFs may return sets of winners, we use tiebreaking procedures to choose a single winner. In our theoretical results, we use lexicographic tiebreaking. In our empirical results, we break ties uniformly at random due to our method of vote completion.

[3]Tiebreaking occurs in each round of RCV.

[4]This occurs when voters submit incomplete preferences.

[5]Head-to-head ties count as half a win.

max chooses the alternative that has the smallest maximum margin of defeat in all head-to-head comparisons.

Bucklin starts with all first-place votes and iteratively adds second-place votes, third-place votes, and so on until an alternative reaches a majority of all votes counted so far; that alternative is returned as the winner. Plurality Veto [Kizilkaya and Kempe, 2022] decrements each alternative's plurality score through n rounds of a veto process, taken in a randomly permuted order, and the last remaining candidate wins.

3 Worst-Case Accuracy of RCV

Somewhat paradoxically, we find that the worst-case accuracy of RCV seems to *decrease* as we increase the size of the sample we take. However, as we will see in the empirical section, this trend is reversed in practice.

Throughout this section, we will analyze RCV with lexicographic (i.e., alphabetical) tiebreaking where, for instance, a_1 defeats a_2 if they are tied.

3.1 Sampling a Single Vote

We begin our analysis in the case of $|s| = 1$, i.e., with samples consisting of only a single vote from the profile. In this case, we can show a tight bound on the probability that RCV predicts itself correctly.

Theorem 1. *For $g(n) = 1$ and $m \geq 2$, $A_R(g, m) = \frac{1}{2^{m-1}}$.*

Proof. We first show the upper bound: $A_R(g, m) \leq \frac{1}{2^{m-1}}$. Consider the following profile:

$$\begin{array}{ll} 2^{m-2} : a_m \succ \dots & \vdots \\ 2^{m-3} : a_{m-1} \succ a_1 \succ \dots & 2 : a_3 \succ a_1 \succ \dots \\ 2^{m-4} : a_{m-2} \succ a_1 \succ \dots & 1 : a_2 \succ a_1 \succ \dots \\ \vdots & 1 : a_1 \succ \dots \end{array}$$

Here, the notation $c : \sigma_i$ means that c voters have the preference σ_i. In this scenario, a_1 wins the election and starts with only 1 vote, which is $\frac{1}{2^{m-1}}$ of the votes in the election. Therefore, RCV predicts itself correctly with probability $\frac{1}{2^{m-1}}$. This profile can be multiplied to create arbitrarily large elections in which this holds.

Now, we show the matching lower bound: $A_R(g, m) \geq \frac{1}{2^{m-1}}$. Note that this proof works for profiles in which all ballots have complete rankings, but it is possible to modify it to work with incomplete rankings as well; see Appendix A.7. Running RCV on a single ballot selects that ballot's first choice as the winner, so the question of finding the worst case probability of RCV predicting itself correctly on a single randomly selected ballot is exactly the same as determining how small we can make the portion of ballots that select the true RCV winner, a^*, as the first choice. Let $v_k(a_i)$ represent the number of first choice votes that alternative a_i receives in round k. For all $k \in [2, m-1]$, we know that $v_k(a_i) \leq 2v_{k-1}(a_i)$ for all a_i not eliminated by round k because the losing candidate of round $k-1$ always has the fewest first choice votes in that round, so any other candidate cannot more than double their first place vote share from one round to the next. We also know that by the final round, i.e., round $m-1$, the RCV winner a^* must have at least half of all votes. Therefore, $v_{m-1}(a^*) \geq n/2$. Now, applying the relation above, we see that $v_1(a^*) \geq \frac{1}{2}v_2(a^*) \geq \dots \geq \frac{1}{2^{m-3}}v_{m-2}(a^*) \geq \frac{1}{2^{m-2}}v_{m-1}(a^*) \geq \frac{n}{2^{m-1}}$, as desired. □

3.2 Sampling All but k Votes

We now move to the other end of the sample size spectrum and ask how well RCV can predict itself given access to almost all of the votes in a profile. Intuitively, it seems like having access to more votes should only help the accuracy of RCV when predicting itself, but we will see that this is not necessarily the case.

Our next theorem states that, even with all but one sample from a profile, RCV's worst-case predictive accuracy is 0, i.e., there exist profiles such that running RCV on any sample of all but one vote returns a different winner than running RCV on the entire profile.

Theorem 2. *For $g(n) = n - 1$ and all $m \geq 4$, we have $A_R(g, m) = 0$.*

Proof. For as few as four candidates, it is possible to construct arbitrarily large profiles in which sampling every ballot but one always yields the incorrect result. Consider the following election:

$$\begin{array}{ll} 2 : a_4 \succ a_1 \succ a_3 \succ a_2 & 2 : a_1 \succ a_4 \succ a_3 \succ a_2 \\ 2 : a_3 \succ a_4 \succ a_2 \succ a_1 & 2 : a_2 \succ a_4 \succ a_3 \succ a_1 \end{array}$$

One can verify that a_1 wins in this profile. Despite this, when we sample all but one vote, if the missing vote has a first choice other than a_4, a_4 ends up winning, and otherwise when we remove a ballot with a_4 as a first choice, a_2 ends up winning. When we scale up this profile to larger sizes by multiplying the number of each ballot by some constant, this property remains, so we can construct arbitrarily large elections in which sampling all but one vote and performing RCV never yields the true winner of the election.

Lastly, in order to extend this construction to $m > 4$, we can add additional candidates in an arbitrary order at the end of each of the votes. □

In fact, we can show a more general statement: Even with all but k samples from a profile for some constant k, we can construct profiles such that RCV's worst-case predictive accuracy is 0. However, m must depend linearly on k.

Theorem 3. *For $g(n) = n - k$ for constant k and all $m \geq 2(k+1)$, we have $A_R(g, m) = 0$.*

Proof. For ease of exposition, we will show an explicit construction for $m = 2(k+1)$, but we can add additional candidates at the end of each vote in the construction without affecting any of the calculations, so the same argument applies for all $m \geq 2(k+1)$.

Let there be $m = 2(k+1)$ candidates in our construction. We will build a profile such that sampling all but k ballots and running RCV always fails to select the true RCV winner on the entire profile. Our profile contains m different types of ballots, each with a different first choice candidate. For ballots with a first choice a_i where $1 \leq i \leq \frac{m}{2}$, the ballot order is $a_i \succ a_m \succ a_{m-1} \succ \cdots \succ a_{i+1}$. For ballots with a first choice a_i where $\frac{m}{2}+1 \leq i \leq m$, the ballot order is a_i, followed by $a_m \succ a_{m-1} \succ \cdots \succ a_{i+1}$ (if $i \neq m$), followed by a_{m-i+1}, followed by $a_{i-1} \succ a_{i-2} \succ \cdots \succ a_{m-i+2}$ (if $i \neq \frac{m}{2}+1$). Finally, there are m copies of each ballot for a total of $n = m^2$ votes.

An example of our construction for $m = 6$ is as follows:

$$\begin{aligned}
6 &: a_6 \succ a_1 \succ a_5 \succ a_4 \succ a_3 \succ a_2 \\
6 &: a_5 \succ a_6 \succ a_2 \succ a_4 \succ a_3 \\
6 &: a_4 \succ a_6 \succ a_5 \succ a_3 \\
6 &: a_3 \succ a_6 \succ a_5 \succ a_4 \\
6 &: a_2 \succ a_6 \succ a_5 \succ a_4 \succ a_3 \\
6 &: a_1 \succ a_6 \succ a_5 \succ a_4 \succ a_3 \succ a_2
\end{aligned}$$

Note that these ballots utilize incomplete rankings. Ballots whose last remaining choice is eliminated are simply removed from the election.

We can verify that a_1 wins in these profiles. The important thing to notice is that there are two halves—the ballots whose first choice is $a_{\frac{m}{2}+1}$ through a_m, which we will call the first half, and those with a first choice a_1 through $a_{\frac{m}{2}}$, the second half. As we eliminate votes from the first half due to lexicographic tie breaking one by one, candidates in the second half gain entire piles of votes from eliminated candidates from the first half. For example, the ballots choosing a_m go to a_1, the ballots choosing a_{m-1} go to a_2, and so on until the ballots choosing $a_{\frac{m}{2}+1}$ go to $a_{\frac{m}{2}}$. During the second half of eliminations, when candidates are eliminated one by one due to lexicographic tie breaking, the votes are simply removed due to the incomplete rankings.

Now, consider sampling all but $k = \frac{m}{2} - 1$ votes from this profile. We will talk about which votes are "removed" from the sample, i.e., the ones not included in the sample. Since each pile of identical votes contains m votes, it is impossible to remove an entire pile, so every round during the first half of eliminations will inevitably result in one candidate gaining votes. We must remove a vote from the ballots that chose a_m as the first choice, because if we don't, a_m will not lose in the first round, and since a_m is the second choice of all of the other kinds of ballots, a_m will gain enough first choice votes from this round to go on to win. In fact, any deviation from the true elimination order will end up giving the highest number candidate remaining a decisive lead and they will go on to win the election, so we must eliminate in the same order as in the complete profile. Eventually we will arrive at the second half of eliminations when $a_{\frac{m}{2}+1}$ is eliminated. Since it is necessary to remove one of the ballots ranking a_m first, and since these ballots go to a_1, a_1 will have lost at least one first choice ballot going into the second half of eliminations. Since there are $\frac{m}{2}$ candidates remaining when we arrive to the second half of eliminations and we removed $\frac{m}{2} - 1$ ballots, it must follow that at least one candidate, let us say a_i, has not lost any first choice votes. This means it is impossible for a_1 to win, as even if we arrive at a_i by eliminating alternatives in the correct order, a_1 will be eliminated before a_i.

These profiles can be scaled up and the above argument still holds, so we can construct arbitrarily large profiles with $m = 2(k+1)$ candidates in which sampling all but k votes will always fail to predict the correct winner. □

We also consider the worst-case performance of RCV on samples that are a constant fraction of the number of voters. In this case, we obtain an upper bound on the worst-case accuracy for RCV of $\frac{1}{m!}$.[6]

Theorem 4. *For $g(n) = \alpha n$ for constant $\alpha \in (0, 1)$ and $m \geq 2$, $A_R(g, m) \leq \frac{1}{m!}$.*

4 Margin of Victory and Sampling Bounds

One additional aspect of sampling we are interested in is the number of samples above which we are *guaranteed* to pick the correct winner. The results in the previous section demonstrate that there exist worst-case profiles that provide no such guarantee until the sample consists of the entire profile. However, all of the worst-case results are balanced on a knife's edge, and changing even one vote can change the winner of the overall election.

Therefore, we analyze these thresholds in terms of the margin of victory of the winning candidate in the entire election, where the margin of victory for profile $\vec{\sigma}$, $M(\vec{\sigma})$, is defined as the total number of votes that have to be modified in order to change the winner of the election. Note that this definition is the same as in [Bhattacharyya and Dey, 2021]. It is also closely related to another definition proposed by Cary [2011] in the context of RCV, $M_C(\vec{\sigma})$, which is the total number of votes that must be added or removed to change the winner.

We first show that our definition of margin of victory, $M(\cdot)$, is related to Cary's definition, $M_C(\cdot)$.

Proposition 1. *For all $\vec{\sigma}$, $\frac{1}{2}M_C(\vec{\sigma}) \leq M(\vec{\sigma}) \leq M_C(\vec{\sigma})$.*

For any profile $\vec{\sigma}$ with margin of victory $M(\vec{\sigma})$, we can also develop upper bounds on the minimum sample size required for RCV to always be correct on any sufficiently large sample; these bounds are illustrated in Figure 1.

Proposition 2. *For a profile $\vec{\sigma}$ consisting of n votes and m candidates, let x be the number of first choice votes for the RCV winner. All samples of size at least $\min(2(n-x)+1, (m-1)(n-2M(\vec{\sigma}))+1)$ are guaranteed to return the correct winner.*

Along the lines of Cary [2011], we may also derive upper and lower bounds on $M(\vec{\sigma})$ that depend on the sequence of eliminations taken by RCV.

Proposition 3. *For a profile $\vec{\sigma}$, $M(\vec{\sigma}) \in$*

$$\left[\left\lceil \frac{1}{2} \min_{k \in [m-1]} \left(v_k^{(-2)} - v_k^{(-1)}\right)\right\rceil, \min_{k \in [m-1]} \left(\left\lfloor \frac{n}{2} \right\rfloor + 1 - v_k^{(2)}\right)\right],$$

where $v_k^{(j)}$ for all $j \in [1, m-k+1]$ is the vote share for the j^{th} most popular alternative in that round, and $v_k^{(-1)}$ and $v_k^{(-2)}$ are the vote shares of the alternatives that receive the fewest and second-fewest votes in round k, respectively.

[6] In Appendix A.5, we describe another setup that, in mathematical simulations, does even worse than $\frac{1}{m!}$.

		RCV	Plurality	Borda	Harm.	Cope.	MM	Bucklin	Pl. Veto	Veto
M, $\phi = 0.5$	ACC	73.60	68.97	72.23	70.37	74.56	74.43	70.10	**97.46**	38.63
	MPW	2.36e-2	1.05e-2	1.50e-2	1.07e-2	2.74e-2	2.67e-2	1.03e-2	8.76e-1	2.59e-5
M, $\phi = 0.75$	ACC	44.15	41.44	46.70	43.86	45.96	44.69	43.30	**78.18**	33.49
	MPW	6.83e-3	4.26e-3	8.60e-3	5.88e-3	7.46e-3	7.75e-3	5.26e-3	9.53e-1	9.98e-4
Urn, $\alpha = 0.05$	ACC	**40.13**	39.50	38.56	39.00	39.34	39.21	37.30	28.86	24.71
	MPW	1.48e-1	1.52e-1	1.12e-1	1.20e-1	1.33e-1	1.37e-1	1.24e-1	4.25e-2	3.13e-2
Conitzer SPOC	ACC	**27.91**	27.18	27.74	27.81	27.50	26.75	26.67	25.18	23.25
	MPW	1.11e-1	1.22e-1	1.14e-1	1.17e-1	9.88e-2	1.02e-1	1.10e-1	1.13e-1	1.12e-1
Walsh SP	ACC	55.44	46.67	**65.73**	56.53	61.13	61.53	47.60	43.94	32.86
	MPW	8.40e-2	1.92e-2	3.55e-1	7.13e-2	2.13e-1	2.13e-1	1.83e-2	2.48e-2	1.48e-3
3D Cube	ACC	47.64	41.84	**56.38**	48.67	51.72	52.61	47.35	35.60	37.60
	MPW	1.09e-1	4.06e-2	3.04e-1	9.79e-2	1.67e-1	1.67e-1	7.61e-2	2.03e-2	1.81e-2
5D Sphere	ACC	31.08	**32.19**	27.99	30.10	28.81	28.91	29.11	26.38	17.11
	MPW	1.31e-1	1.58e-1	8.70e-2	1.36e-1	9.61e-2	1.02e-1	1.15e-1	1.15e-1	6.10e-2

Table 1: Accuracy of various rules predicting the RCV winner along with normalized multiplicative weights for 5% sample sizes. Rows marked "ACC" are accuracies in percents, and rows marked "MPW" are the learnt multiplicative weights.

		RCV	Plurality	Borda	Harm.	Cope.	MM	Bucklin	Pl. Veto	Veto
M, $\phi = 0.5$	ACC	99.49	98.50	99.09	99.06	99.32	99.31	79.11	**1.000**	77.98
	MPW	1.47e-1	1.31e-1	1.34e-1	1.38e-1	1.46e-1	1.46e-1	3.53e-3	1.52e-1	1.76e-3
M, $\phi = 0.75$	ACC	82.30	75.00	79.64	79.41	83.01	82.33	69.85	**88.00**	62.25
	MPW	1.50e-1	2.15e-2	1.20e-1	6.38e-2	1.54e-1	1.73e-1	1.23e-2	3.02e-1	2.74e-3
Urn, $\alpha = 0.05$	ACC	**70.85**	64.05	63.02	69.64	66.65	69.04	45.57	16.07	33.42
	MPW	2.73e-1	7.34e-2	1.03e-1	1.65e-1	1.67e-1	2.09e-1	9.43e-3	4.88e-5	7.97e-4
Conitzer SPOC	ACC	**49.46**	44.92	44.08	47.31	43.11	42.64	29.48	19.00	29.92
	MPW	2.17e-1	1.49e-1	1.42e-1	1.92e-1	1.32e-1	1.21e-1	1.72e-2	8.06e-3	2.15e-2
Walsh SP	ACC	83.55	79.78	**88.01**	86.17	85.84	87.87	53.33	3.770	33.16
	MPW	7.62e-2	4.16e-2	2.22e-1	2.11e-1	2.24e-1	2.26e-1	2.78e-4	8.95e-8	6.89e-6
3D Cube	ACC	**77.78**	60.42	74.82	74.48	75.16	77.57	66.05	23.22	53.99
	MPW	2.16e-1	1.14e-2	1.49e-1	1.49e-1	2.20e-1	2.34e-1	1.90e-2	5.40e-6	2.49e-3
5D Sphere	ACC	82.15	74.15	81.43	82.57	82.52	**82.95**	72.38	25.95	10.41
	MPW	2.40e-1	3.10e-1	4.11e-2	2.68e-1	5.67e-2	6.64e-2	1.35e-2	3.69e-3	1.30e-3

Table 2: Accuracy of various rules predicting the RCV winner along with normalized multiplicative weights for 50% sample sizes. Rows marked "ACC" are accuracies in percents, and rows marked "MPW" are the learnt multiplicative weights.

5 Experiments

In our experiments, we explore two main questions on a mix of synthetic elections generated from statistical cultures in the map of elections [Szufa *et al.*, 2020; Boehmer *et al.*, 2021] and real-world election data sourced from PrefLib [Mattei and Walsh, 2013] and the Harvard Dataverse [Harvard, 2020]. First, on synthetic elections, we examine the prediction accuracy of various voting rules when predicting the RCV winner on uniform samples of varying sizes. Second, on real-world elections, we examine the accuracy with which the RCV winner can be correctly predicted by each of the voting rules we consider, as well as two additional "ensemble" rules informed by results on the map of elections. Our code is available at https://github.com/miceland2/STV_sampling.

5.1 Synthetic Elections

Informed by prior work on the map of elections, we use the `mapel` Python library to generate votes from the diverse set of statistical cultures included in the original map. These include the Mallows model (with dispersion parameter $\phi \in \{0.001, 0.01, 0.05, 0.1, 0.25, 0.5, 0.75, 0.95, 0.99, 0.999\}$), Polya-Eggenberger urn models (with $\alpha \in \{0.01, 0.02, 0.05, 0.1, 0.2, 0.5\}$), the Conitzer and Walsh single-peaked models, the Conitzer single-peaked on a circle (SPOC) model, single-crossing models, the Impartial Culture model, 1D, 2D, 3D, 5D, 10D, and 20D hypercube models, and finally 2D, 3D, and 5D hypersphere models. In the interest of space, for further discussion of the specific statistical cultures in these models, see Section 2.2 in [Szufa *et al.*, 2020].

In our experiments, we vary the sample size from 10% to 100% in steps of 10%, with the addition of a 5% sample size; additional results can be found in Appendix B.3.

In Tables 1 and 2, we present the average prediction accuracy ("ACC") of each of our nine voting rules when predicting the RCV winner for profiles generated according to the statistical cultures we consider for samples consisting of 5% and 50% of the voters, respectively. The average prediction accuracy is taken over 100 samples on each of 100 different profiles generated according to the statistical cultures in consideration. These profiles each consist of 100 votes over 5

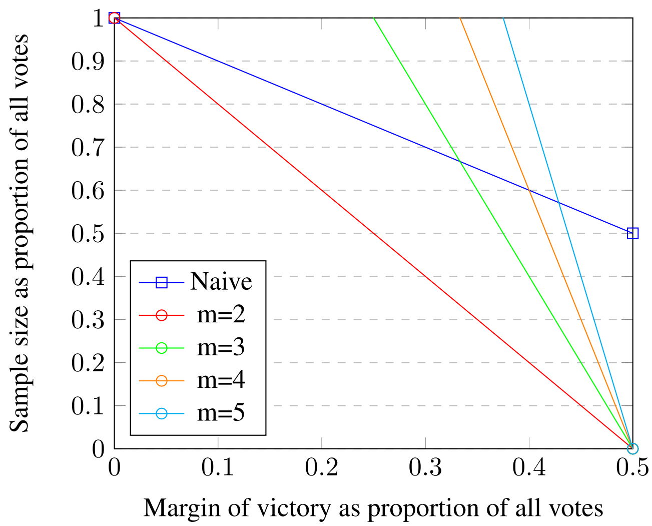

Figure 1: Bounds on the minimum sample size needed to ensure that evaluating RCV on any sample will identify the correct winner. For each m, running RCV with any sample size above the corresponding line is guaranteed to return the correct winner.

alternatives, which is roughly the average number of candidates over all our real-world data.

For each statistical culture, we also treat each voting rule as an expert and learn normalized weights for each voting rule via the classic multiplicative weights ("MPW") algorithm [Littlestone and Warmuth, 1994; Arora *et al.*, 2012].

Overall, we find that RCV exhibits uneven performance across different statistical vote cultures, but its accuracy increases with sample size. Plurality Veto is unexpectedly accurate for Mallows models, as evinced by its remarkably high MPW score in these settings. On the whole, we observe that more "centered" distributions like Mallows are generally easier to predict than other families, most likely due to RCV's sensitivity to the order of eliminations in scenarios without a clear majority winner.

5.2 Real-World Elections

One of our main empirical questions is whether we can leverage results from synthetic vote profiles to achieve greater prediction accuracy on real-world elections. To this end, we build two ensemble methods that leverage the pseudo-distance metric underlying the map of elections in order to predict the RCV winner of real-world elections.

The central idea behind these ensemble methods is to use good predictors of RCV on "nearby" elections on the map of elections in order to predict RCV outcomes on real-world data. Given a sample s, both ensemble methods first identify the closest statistical culture according to positionwise distance as defined in Section 3.2 in [Szufa *et al.*, 2020]. We call the closest statistical culture $\mathcal{C}$, and use our empirical results on $\mathcal{C}$ to create scores for each alternative. Throughout, let $\mathcal{R}$ represent the set of rules we define in Section 2.1.

The first ensemble method, which we term "Summation," uses our experiments on synthetic data and adds $acc_f(\mathcal{C})$, which we define as the empirical accuracy of rule f predicting RCV on culture $\mathcal{C}$, to the score of the winner $f(s)$ for each rule $f \in \mathcal{R}$. The alternative with the highest overall score after this process is the Summation winner.

The second ensemble method, which we call "MPW," selects a predictive rule to use according to a probability distribution based on the normalized weights learned on $\mathcal{C}$ through the multiplicative weights process. The alternative returned by the predictive rule is the MPW winner.

We run experiments to measure the performance of the nine rules in Section 2.1, as well as these two ensemble rules, on a total of 12 collections of different real-world elections from PrefLib [Mattei and Walsh, 2013] and Harvard Dataverse [Harvard, 2020], amounting to a total of 275 individual elections. Each collection consists of between 8 and 46 separate elections, each of which contain between 143 and 39,401 votes on 2 to 15 candidates. Full descriptions can be found in Appendix B.1.

Preprocessing

For each dataset in the Harvard database, we take all available profiles of the locality and government position. We exclude elections that consist of a single candidate. For each profile, we (1) discard blank rows, (2) remove table cells labeled "write-in," "overvote," or "skipped," and (3) keep only the higher-ranked position for each vote if the voter gave two or more rankings of the same alternatives. Generally, the final preprocessing step applied to less than 10% of all votes for each profile. In contrast, datasets from Preflib did not require the preprocessing steps described above.

All real-world elections give strict-order-incomplete rankings over the candidates, where unranked candidates in a given vote are assumed to be tied for last place. We complete each of these incomplete rankings using the same method proposed by Boehmer *et al.* [2021] in order to (1) run each of our voting rules without modifications or additional assumptions and (2) compute the positionwise distances between each real election and those from the map of elections using the `mapel` library. For each vote v that gives an incomplete ranking for their top t candidates, we first draw uniformly at random another vote that ranks at least the top $(t+1)$ candidates and agrees with v on the top t candidates, and we then extend v with this other vote's $(t+1)^{st}$-ranked candidate. If no such vote exists, we extend v with one of their unranked candidates uniformly at random. The process is repeated until all votes are strict-order complete.

Real-World Results

We present the average RCV sampling accuracies for each of our rules for various collections of elections in Figure 2 and Appendix B.4. For each sample size (5%, 10%, 30%, 50%, 70%, and 100%) and each election, we estimate the sampling accuracy with 1,000 samples. The right-most column contains average accuracies for all elections in each group, and the other two columns show results from individual elections in each group. The center column contains plots that are more typical of the dataset, while those on the left are more extraneous and typically have lower bounds on the margin of victory.

We observe that, in contrast to the theoretical results and results on synthetic data, RCV is on average one of the best predictors of itself even on low sample sizes. While this does

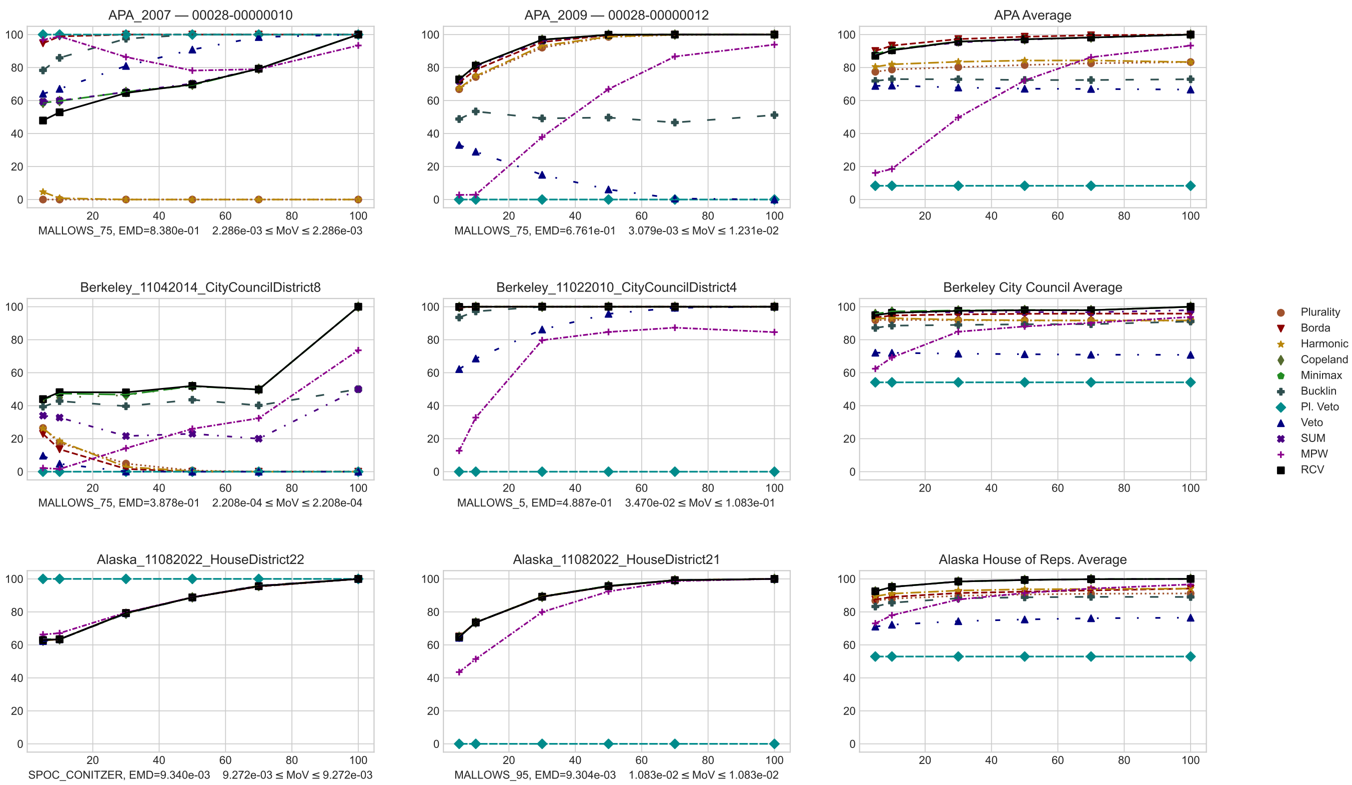

Figure 2: Summary and individual plots for the APA, Berkeley City Council, and Alaska House of Representatives datasets. We show the closest statistical culture and bounds on the MoV for individual elections. EMD is the positionwise distance [Szufa *et al.*, 2020].

not mean that RCV is the best predictor of itself on every individual election, on average, this trend persists over all our real-world data. Only among the Glasgow City Council elections, though, is RCV decisively the best predictor. Summation performs almost as well as RCV on average. This is likely because the ensemble rules tend to agree with RCV on high sample sizes for all the real-world data we considered, whereas other voting rules sometimes diverged from the RCV winner as sample size increases. The Condorcet-consistent rules, namely Copeland and Minimax, are also among the best predictors of RCV and only rarely diverge from the true winner. On the other hand, MPW often suffers from poor performance on low sample sizes before catching up at higher sample sizes. This is due to the fact that, as seen in Tables 1 and 2, Plurality Veto has a very high weight in small samples for Mallows elections; its weight decreases as sample size increases. However, Plurality Veto often does very poorly in predicting the overall RCV winner in practice. Although on some profiles, such as those in the top-left and bottom-left of Figure 2, Plurality Veto is an exceptional predictor of RCV even on 5% sample sizes, such profiles are not common, and Plurality Veto often does not increase in accuracy as sample size increases.

We also note that, in direct contrast to our worst-case results, the average predictive performance of RCV increases with sample size, corroborating prior observations that real-world elections are far from the worst-case profiles we study [Boehmer and Schaar, 2023].

Finally, we conclude that the positionwise distance is limited in its ability to extrapolate sampling behavior from one election to a nearby election in terms of positionwise distance. The most obvious evidence comes from the disparity in performance of Plurality Veto between the synthetic Mallows profiles and the real-world elections. As seen in Figure 2, Plurality Veto is by far the worst, on average, at predicting RCV for all three datasets, yet most of their profiles are closest to one of the Mallows cultures. This disparity in performance can be explained by the fact that a given positionwise frequency matrix can map to several different profiles, as explored in [Boehmer *et al.*, 2023].

6 Discussion

This paper presents a theoretical and empirical exploration of using RCV on samples to predict the outcome of applying RCV on the entire election. We establish that, while RCV exhibits bad worst-case theoretical accuracy, it is generally the most trustworthy predictor of itself in practice.

As for future work, there are two main avenues to pursue. With respect to theoretical results, it would be interesting to fully characterize the conjectured monotonicity of the worst-case prediction accuracy of RCV. We present some initial results toward this goal in Appendix A.6. We also will study average-case predictability of RCV on samples instead of worst-case predictability. On the empirical side, we plan to extend our analysis to additional real-world voting data and study more theoretically sound ensemble rules.

7 Acknowledgments

The authors would like to thank David McCune for his helpful comments on STV and for pointing us to the Harvard Dataverse, as well as anonymous AAAI reviewers for their thoughtful suggestions.

References

[Arora *et al.*, 2012] Sanjeev Arora, Elad Hazan, and Satyen Kale. The multiplicative weights update method: a meta-algorithm and applications. *Theory of computing*, 8(1):121–164, 2012.

[Bhattacharyya and Dey, 2021] Arnab Bhattacharyya and Palash Dey. Predicting winner and estimating margin of victory in elections using sampling. *Artificial Intelligence*, 296:103476, 2021.

[Boehmer and Schaar, 2023] Niclas Boehmer and Nathan Schaar. Collecting, classifying, analyzing, and using real-world elections. In *Proceedings of the 22nd International Conference on Autonomous Agents and Multi-Agent Systems (AAMAS)*, pages 1706–1715, 2023.

[Boehmer *et al.*, 2021] Niclas Boehmer, Robert Bredereck, Piotr Faliszewski, Rolf Niedermeier, and Stanisław Szufa. Putting a compass on the map of elections. In *Proceedings of the 30th International Joint Conference on Artificial Intelligence (IJCAI)*, pages 59–65, 2021.

[Boehmer *et al.*, 2023] Niclas Boehmer, Jin-Yi Cai, Piotr Faliszewski, Austen Z Fan, Łukasz Janeczko, Andrzej Kaczmarczyk, and Tomasz Was. Properties of position matrices and their elections. In *The 37th AAAI Conference on Artificial Intelligence (AAAI)*, pages 5507–5514, 2023.

[Brandt *et al.*, 2016] Felix Brandt, Vincent Conitzer, Ulle Endriss, Jérôme Lang, and Ariel D Procaccia. *Handbook of computational social choice*. Cambridge University Press, 2016.

[Cary, 2011] David Cary. Estimating the margin of victory for instant-runoff voting. In *2011 Electronic Voting Technology Workshop/Workshop on Trustworthy Elections (EVT/WOTE 11)*, 2011.

[Dey and Narahari, 2015] Palash Dey and Yadati Narahari. Estimating the margin of victory of an election using sampling. In *Proceedings of the 24th International Joint Conference on Artificial Intelligence (IJCAI)*, pages 1120–1126, 2015.

[Donovan *et al.*, 2019] Todd Donovan, Caroline Tolbert, and Kellen Gracey. Self-reported understanding of ranked-choice voting. *Social Science Quarterly*, 100(5):1768–1776, 2019.

[Faliszewski *et al.*, 2019] Piotr Faliszewski, Piotr Skowron, Arkadii Slinko, Stanisław Szufa, and Nimrod Talmon. How similar are two elections? In *Proceedings of the AAAI Conference on Artificial Intelligence*, volume 33, pages 1909–1916, 2019.

[Felsenthal and Tideman, 2013] Dan S Felsenthal and Nicolaus Tideman. Varieties of failure of monotonicity and participation under five voting methods. *Theory and Decision*, 75:59–77, 2013.

[Graham-Squire and McCune, 2023] Adam Graham-Squire and David McCune. An examination of ranked-choice voting in the united states, 2004–2022. *Representation*, pages 1–19, 2023.

[Harvard, 2020] Harvard. Harvard Dataverse. https://dataverse.harvard.edu/dataset.xhtml?persistentId=doi:10.7910/DVN/AMK8PJ, 2020. Accessed: 2023-09-06.

[Horton and Thomas, 2023] Jennifer Horton and Dakota Thomas. Ranked choice voting: What, where, why & why not, 2023.

[Kizilkaya and Kempe, 2022] Fatih Erdem Kizilkaya and David Kempe. Plurality veto: A simple voting rule achieving optimal metric distortion. arXiv preprint arXiv:2206.07098, 2022.

[Littlestone and Warmuth, 1994] Nick Littlestone and Manfred K Warmuth. The weighted majority algorithm. *Information and computation*, 108(2):212–261, 1994.

[Mattei and Walsh, 2013] Nicholas Mattei and Toby Walsh. Preflib: A library for preferences http://www. preflib. org. In *Algorithmic Decision Theory: Third International Conference, ADT 2013, Bruxelles, Belgium, November 12-14, 2013, Proceedings 3*, pages 259–270. Springer, 2013.

[Micha and Shah, 2020] Evi Micha and Nisarg Shah. Can we predict the election outcome from sampled votes? In *Proceedings of the 34th AAAI Conference on Artificial Intelligence (AAAI)*, pages 2176–2183, 2020.

[Spencer *et al.*, 2015] Andrew Spencer, Christopher Hughes, and Rob Richie. Escaping the thicket: The ranked choice voting solution to america's redistricting crisis. *Cumb. L. Rev.*, 46:377, 2015.

[Szufa *et al.*, 2020] Stanislaw Szufa, Piotr Faliszewski, Piotr Skowron, Arkadii M. Slinko, and Nimrod Talmon. Drawing a map of elections in the space of statistical cultures. In *Proceedings of the 19th International Conference on Autonomous Agents and Multi-Agent Systems (AAMAS)*, pages 1341–1349, 2020.

[Tolbert and Kuznetsova, 2021] Caroline Tolbert and Daria Kuznetsova. Editor's introduction: The promise and peril of ranked choice voting. *Politics and Governance*, 9(2):265–270, 2021.

A Theoretical Results

In this section, we present the proofs of Theorem 4, Proposition 1, Proposition 2, and Proposition 3, as well as additional upper bounds on the worst-case accuracy for both RCV and other rules when predicting RCV.

A.1 Proof of Theorem 4

Theorem 4. *For $g(n) = \alpha n$ for constant $\alpha \in (0, 1)$ and $m \geq 2$, $A_R(g, m) \leq \frac{1}{m!}$.*

Proof. We define the profiles A_i for $i \geq 1$. Each A_i consists of a profile of size $i!$ over i candidates. A_1 is a profile consisting of a single ballot voting for a_1.

Each A_i is defined recursively as follows. For each $j < i$, for every ballot in A_{i-1} ranking a_j first, we add $i - 1$ new ballots to A_i which are the same as the ballot except that we insert a_i as the second choice and slide the remaining rankings down. Additionally, for every ballot in A_{i-1}, we add a new ballot to A_i which is the same except that they rank a_i first and slide the remaining preferences down.

Some examples for small m:

$m = 2$:

$$1: a_1 \succ a_2 \qquad\qquad 1: a_2 \succ a_1$$

$m = 3$:

$$\begin{array}{ll} 2: a_1 \succ a_3 \succ a_2 & \qquad 2: a_2 \succ a_3 \succ a_1 \\ 1: a_3 \succ a_1 \succ a_2 & \qquad 1: a_3 \succ a_2 \succ a_1 \end{array}$$

$m = 4$:

$$\begin{array}{ll} 6: a_1 \succ a_4 \succ a_3 \succ a_2 & \qquad 6: a_2 \succ a_4 \succ a_3 \succ a_1 \\ 3: a_3 \succ a_4 \succ a_1 \succ a_2 & \qquad 3: a_3 \succ a_4 \succ a_2 \succ a_1 \\ 2: a_4 \succ a_1 \succ a_3 \succ a_2 & \qquad 2: a_4 \succ a_2 \succ a_3 \succ a_1 \\ 1: a_4 \succ a_3 \succ a_1 \succ a_2 & \qquad 1: a_4 \succ a_3 \succ a_2 \succ a_1 \end{array}$$

We can verify that a_1 wins in this setup: during each round, there is a tie between all remaining candidates and the highest number candidate is removed by lexicographic tie breaking. Candidates are eliminated from highest number to least, i.e. from a_m to a_1.

These profiles can be multiplied for larger and larger elections. When a candidate is eliminated, the setup recurses one lower, so the current profile which is a multiple of the A_i profile becomes a multiple of the A_{i-1} profile.

As the election grows larger and the fraction from which we sample remains constant, each candidate will receive very nearly $1/m$ of the first choice votes. In the complete election, the elimination order is from a_m to a_1, i.e. a_m is eliminated in the first round, then a_{m-1} in the next round, and so on until a_1 is eliminated in the final round. In this profile, the highest number candidate is always the second choice of all of the ballots that did not rank them first. Because of this, if we eliminate any candidate other than the highest number candidate, all of the votes will go to the highest number candidate. For large elections, this will with very high probability put the highest number candidate above all others by a substantial margin, and she will go on to win the election.

Because of this, for any sample which also chooses a_1 as the winner, it is necessary to follow the same elimination order as in the election on the complete profile, and wins that do not take this order will make up a vanishing portion for larger and larger profiles.

By symmetry, each candidate is equally likely to lose not due to tie breaking in the first round, and losses due to tiebreaks will make up a vanishing portion for large elections, so we have a $\frac{1}{m}$ chance of eliminating a_m in the first round. We can see that among the ballots that rank a_m as the first choice, the second choice preferences are evenly distributed across the remaining candidates, so there is still a symmetry for the remaining candidates, giving us a $\frac{1}{m-1}$ chance of eliminating a_{m-1} next. Continuing like this, we can see in the third round we have a $\frac{1}{m-2}$ chance of making the correct elimination, and so on until the final round when we have a $\frac{1}{2}$ chance. Putting this all together gives us a $\frac{1}{m!}$ chance of choosing a_1 as the winner in the sample. □

If we choose to leverage the power of incomplete rankings, we can achieve the $\frac{1}{m!}$ bound from before with a much simpler setup.

We describe the profiles A_i, each being an a profile with i candidates. For every $1 \leq j \leq i$, we add one ballot to A_i with a first choice of a_j followed by a_i to a_{j+1} in descending order. Here are some examples:

$m = 2$

$$1:\ a_1 \succ a_2 \qquad\qquad 1:\ a_2$$

$m = 3$

$$1:\ a_1 \succ a_3 \succ a_2 \qquad\qquad 1:\ a_2 \succ a_3$$
$$1:\ a_3$$

$m = 4$

$$1:\ a_1 \succ a_4 \succ a_3 \succ a_2 \qquad\qquad 1:\ a_2 \succ a_4 \succ a_3$$
$$1:\ a_3 \succ a_4 \qquad\qquad 1:\ a_4$$

In these profiles, a_1 wins in the entire profile and the elimination order is always a_i to a_1 descending. As we multiply the profile to larger sizes, any deviation from this elimination order will nearly guarantee that the highest number candidate remaining will win, as this will give them substantially more votes and they will be the second choice of all ballots that do not rank them first during that round. This in essence requires that the sizes of the piles from least to greatest to be a_i, a_{i-1}, ... a_2, a_1. Again, ties become unlikely for larger elections. Since all of the piles are the same size and there is nothing distinguishing them, this is a $\frac{1}{m!}$ chance, as that is the number of permutations of the piles, each of which can be a potential ordering of their sizes. This setup is substantially simpler, using only a linear number of distinct ballots in i whereas the setup in the main paper uses an exponential number of distinct ballots.

A.2 Proof of Proposition 1

Proposition 1. *For all $\vec{\sigma}$, $\frac{1}{2}M_C(\vec{\sigma}) \leq M(\vec{\sigma}) \leq M_C(\vec{\sigma})$.*

Proof. Since a change in one vote is exactly one addition and one removal, we have that $M_C(\vec{\sigma})$ is never more than twice $M(\vec{\sigma})$, which yields the first inequality, i.e., $\frac{1}{2}M_C(\vec{\sigma}) \leq M(\vec{\sigma})$.

Now, for an election $\vec{\sigma}$ consisting of n votes, consider the fewest additions and removals that results in a different winner, and let x be the number of additions and y be the number of removals. Therefore, $M_C(\vec{\sigma}) = x + y$. We will show how to create another election $\vec{\sigma}'$ of size n that changes at most $\max(x, y) \leq M_C(\vec{\sigma})$ votes from $\vec{\sigma}$ with a different winner.

If $y > x$, this means that we have effectively changed x votes and removed $y - x$ votes from $\vec{\sigma}$. Now, add back $y - x$ first choice votes for the new winner. These will not change the winner, and we have constructed an election the same size as the original with a different winner with a total of at most y changes.

If $x > y$, this means that we have effectively changed y votes and added $x - y$ votes to $\vec{\sigma}$. Now, we can remove $x - y$ votes from the election as follows. We will remove first choice votes for the original winner in $\vec{\sigma}$, and then arbitrary votes once these run out. If there are no first choice votes for the original winner left in $\vec{\sigma}'$, then the winner must change. Otherwise, removing first choice votes for the original winner cannot help them and can only cause them to be eliminated at least as early as they were already eliminated in $\vec{\sigma}'$. Therefore, we have constructed an election the same size as the original with a different winner with a total of at most x changes.

Putting together the two cases yields $M(\vec{\sigma}) \leq \max(x, y) \leq M_C(\vec{\sigma})$, as desired. □

A.3 Proof of Proposition 2

Proposition 2. *For a profile $\vec{\sigma}$ consisting of n votes and m candidates, let x be the number of first choice votes for the RCV winner. All samples of size at least $\min(2(n-x)+1, (m-1)(n-2M(\vec{\sigma}))+1)$ are guaranteed to return the correct winner.*

Proof. We begin with the naive bound of $2(n - x) + 1$, which does not take into account $M(\vec{\sigma})$ or m. The argument is immediate: in all samples of size at least $2(n - x) + 1$, the number of first choice votes for the true winner must be at least half of the size of the sample. However, this is only a useful bound when $x > n/2$.

Now, we will show that all samples of size at least $(m - 1)(n - 2M(\vec{\sigma})) + 1$ are also guaranteed to return the true winner. If there are x first choice votes for the winning candidate, this means that there are $n - x$ ballots with first choice votes for the remaining $m - 1$ candidates. By the pigeonhole principle, one candidate must have at least $\frac{n-x}{m-1}$ first choice votes. Any candidate with more than half of the first choice votes will win, so we must change at most $\frac{n}{2} - \frac{n-x}{m-1}$ votes to change the winner of the election. Therefore, $M(\vec{\sigma}) \leq \frac{n}{2} - \frac{n-x}{m-1}$. Solving for x, we have that $x \geq M(\vec{\sigma})(m-1) - \frac{n(m-3)}{2}$. Substituting this value for x into our naive bound of $2(n - x) + 1$ above yields $(m - 1)(n - 2M(\vec{\sigma})) + 1$, as desired. Note that this bound is sometimes much better than the first, but as m increases, the area in which it beats the naive bound becomes increasingly restricted. □

A.4 Proof of Proposition 3

Proposition 3. *For a profile $\vec{\sigma}$, $M(\vec{\sigma}) \in$*

$$\left[\left\lceil \frac{1}{2} \min_{k\in[m-1]} \left(v_k^{(-2)} - v_k^{(-1)}\right)\right\rceil, \min_{k\in[m-1]} \left(\left\lfloor \frac{n}{2} \right\rfloor + 1 - v_k^{(2)}\right)\right],$$

where $v_k^{(j)}$ for all $j \in [1, m-k+1]$ is the vote share for the j^{th} most popular alternative in that round, and $v_k^{(-1)}$ and $v_k^{(-2)}$ are the vote shares of the alternatives that receive the fewest and second-fewest votes in round k, respectively.

Proof. We begin with the lower bound. In order for the winner to change, there must be a change in some elimination. The fewest number of changes needed to potentially change an elimination in round k is $\left\lceil \frac{1}{2} \cdot \left(v_k^{(-2)} - v_k^{(-1)}\right)\right\rceil$, so

$$M(\vec{\sigma}) \geq \left\lceil \frac{1}{2} \cdot \min_{k\in[m-1]} \left(v_k^{(-2)} - v_k^{(-1)}\right)\right\rceil.$$

To show the upper bound, note that in every round $k \in [m-1]$, if we flip enough votes to the second-place candidate such that the second-place candidate attains more than half of all votes, the overall winner is guaranteed to change. Therefore,

$$M(\vec{\sigma}) \leq \min_{k\in[m-1]} \left(\left\lfloor \frac{n}{2} \right\rfloor + 1 - v_k^{(2)}\right),$$

completing the proof. □

A.5 Worse Than $1/m!$ Upper Bound for RCV

We find that, for $m \geq 4$, there are also setups which can do worse than $\frac{1}{m!}$ with m candidates, which we describe below. However, we found and verified these setups via numerical analysis, and leave proofs of these results to future work.

First, we will consider an additional profile with four candidates that achieves $\frac{1}{24}$ and then we will modify it slightly to achieve $\frac{1}{48}$.

Consider the following profile:

$$1: a_4 \succ a_1 \succ a_3 \succ a_2 \qquad 1: a_3 \succ a_4 \succ a_2 \succ a_1$$
$$1: a_2 \succ a_4 \succ a_3 \succ a_1 \qquad 1: a_1 \succ a_4 \succ a_3 \succ a_2$$

Now we multiply this profile up to a sufficient size so it becomes likely that we receive votes from each profile in roughly equal proportions and so that losses due to tiebreaks and other coincidences such as the size of piles in our sample being equal become highly unlikely.

We must eliminate a_4 first, otherwise, a_4 will go on to dominate the election. This is a $\frac{1}{4}$ chance. Next, either a_2 or a_3 loses in the next round and gives their vote shares to the other. a_1 will not lose in the second round due to being far ahead due to the votes received from a_4. Suppose a_2 wins in this round. If we order the size of each pile from least to greatest, at this point, there are three possibilities: (1) a_4, a_1, a_3, a_2, (2) a_4, a_3, a_1, a_2, or (3) a_4, a_3, a_2, a_1.

Since all we know about the a_1 pile at this point is that it is larger than the a_4 pile, each of these possibilities are equally likely. If we are in one of the first two cases, then barring ties which become unlikely for large profiles the size of the a_4 and a_1 piles combined must be less than the size of the a_3 and a_2 piles combined, so a_1 cannot win in the final round. So we need the final scenario to be the case, and this is a $\frac{1}{3}$ chance. Finally, the size of the a_4 and a_1 piles combined must be greater than the size of the a_3 and a_2 piles combined, which is, we suspect, a $\frac{1}{2}$ chance. If instead, a_3 had beaten a_2, then with switching a_2 and a_3 the same logic as above applies still. Multiplying these probabilities together, there is a $\frac{1}{24}$ chance that a_1 wins.

Now consider the modified profile:

$$k: a_4 \succ a_1 \succ a_3 \succ a_2 \qquad 1: a_4 \succ a_3 \succ a_2 \succ a_1$$
$$1: a_4 \succ a_2 \succ a_3 \succ a_1 \qquad k: a_3 \succ a_4 \succ a_2 \succ a_1$$
$$2: a_3 \succ a_4 \succ a_1 \succ a_2 \qquad k+2: a_2 \succ a_4 \succ a_3 \succ a_1$$
$$k+2: a_1 \succ a_4 \succ a_3 \succ a_2$$

We can see that a_1 still wins in this profile, and that the elimination order is the same as before.

The variable k can be increased as much as we want, which in combination with multiplying the profile to a larger size will decrease a_1's chance of winning. The advantage of this setup is that giving a small fraction of a_4's votes to a_2 and a_3 allows us to make the election choose the wrong winner if a_3 beats a_2 in the second round. In order for us to do this, the a_2 and a_3 votes must make up over half of the profile. The downside is that by the third round if the elimination order proceeds as it did in the complete profile we will have given some of the a_4 votes to a_2 and some of the a_3 votes to a_1, which slightly increases

a_1's chances of winning. However, as we increase k to be bigger and bigger along with the size of the profile, this becomes negligible. This ultimately gives us a $\frac{1}{48}$ chance of choosing a_1 correctly.

We can similarly construct profiles with 6 and 8 candidates that perform worse than the $\frac{1}{m!}$ bound established earlier. These profiles make use of the random variance in the size of the first choice vote shares and incorporate probabilities that size of one combination of piles exceeds the size of another. As we have seen for four candidates, a_1 winning relies on the probability that the largest and smallest piles exceeds the size of the middle. With six candidates, we can create a profile in which a_1 winning relies on the probability that the 1st, 5th, and 6th largest piles exceeds the size of the 2nd, 3rd, and 4th piles. With eight candidates, it is the probability that the 1st, 6th, 7th, and 8th exceeds the 2nd, 3rd, 4th and 5th.

If we let n_k be the size of the kth largest pile, we can write the equation for 6 candidates as: $n_4 + n_3 + n_2 \leq n_6 + n_5 + n_1$, which can be rewritten as $(n_4 - n_6) + (n_3 - n_5) \leq (n_1 - n_2)$. Similarly the equation for the eight candidate setup can be written as $(n_5 - n_8) + (n_4 - n_7) + (n_3 - n_6) \leq (n_1 - n_2)$. In other words, the size of the gap between the first and second candidate's pile must exceed the sum of the sizes of several presumably larger gaps. This suggests that probability decreases very rapidly in m, although we are not sure yet exactly how quickly it decreases and how low a probability we can achieve.

A.6 Bounds for Large Constant Sample Sizes

Interestingly, our constructions used in the proof of Theorem 3—i.e., in the case of sampling $n - k$ ballots, when k is odd and less then $\frac{m}{2}$—still work when we have only 2 copies of each ballot instead of m copies. This allows us to construct arbitrarily large elections in which sampling over 75% of all ballots guarantees an incorrect result with RCV, given the parity of the number of ballots excluded is odd. However, this construction requires linearly many candidates in the size of the profile, so it does not fit nicely into our framework that we have developed thus far. We will now argue that this construction works.

We will organize votes by their original first choice, so we will refer to all the votes with a first choice of a_2 as the a_2 pile. We will refer to votes from a_m to $a_{m/2+1}$ as being on the first half, since in the full election they are eliminated first. We will refer to a pile a_i being "before" a_j if $i > j$, again referencing the original elimination order. We will refer to the candidates a_{m-i} and a_{i+1} (e.g., a_m and a_1, or a_{m-1} and a_2) as "corresponding". This references an important aspect of the elimination order: If a_{i+1} is still in the election, then the a_{m-i} pile's ballots (as long as i is small enough so that a_{m-i} is in the first half) will go to a_{i+1} after all candidates before a_{m-i} are eliminated; otherwise, it will go to the highest-numbered candidate remaining.

For each pile, we have three options: do nothing, remove one vote, or remove both votes. Suppose we do not remove a vote from the a_m pile. Since we are removing an odd number of votes, we must remove exactly one vote from at least one pile. So after we eliminate all the candidates with 0 first choice votes, we eliminate the candidates with only one first choice vote next. Because of the construction, all of these votes go to a_m, and Since a_m is the second choice of all of the other ballots, they will go on to win the election. This means we must remove either one or both votes from the a_m pile.

Now we take notice of several important facts: The only candidate that can have more than four first-choice-votes in any round is the highest number candidate remaining. Besides the highest number candidate remaining, only candidates on the second half can receive more than two first choice votes in any round; this is when their corresponding candidate on the second half loses and gives them their votes. They can in fact, receive up to, but not necessarily, four votes total. Because we have to remove at least one vote from the a_m pile, a_1 can only ever have up to three first-choice votes total in any round. This means if any candidate ever receives more than three first choice votes in any round, it will be impossible for a_1 to win.

Let a_{m-x} be the first candidate whose pile and corresponding candidate's pile on the other half are both undiminished. Such a candidate must exist; since $k < \frac{m}{2}$, it is impossible to remove a vote from all sets of corresponding piles. Clearly, a_{m-x} must be on the first half or we would just select the corresponding candidate instead. If a_{m-x} is never eliminated, then a_{m-x} won the election. Otherwise, there is a round in which a_{m-x} is eliminated. If candidates preceding a_{m-x} are still in the election, then they must have at least three first choice votes, otherwise we would not be eliminating a_{m-x} this round. Then when a_{m-x} is eliminated, the two votes in their pile will go to the highest number candidate remaining, giving them at least five votes. If no candidates preceding a_{m-x} are in the election and a_{x+1} has not been eliminated, the corresponding candidate a_{x+1} will receive the votes and obtain four first-place votes. If no candidates preceding a_{m-x} are in the election and a_{x+1} has been eliminated, then the a_{x+1} pile is currently given to a_{m-x}, the highest-numbered candidate remaining, so they have four first choice votes. The fact that we are eliminating a candidate with four first choice votes means a_1 must have already been eliminated. Thus no matter what, a candidate other than a_1 will receive more than four first choice votes during a round, while a_1 can only ever get at most three, making it impossible for a_1 to win in these samples.

A.7 Extension of Theorem 1 to Incomplete Rankings

We can modify the proof of Theorem 1 to work for profiles in which some ballots give incomplete rankings of the candidates. When a candidate is eliminated and some votes ranking them first have no further rankings, these votes are simply removed. This can affect the portions of candidates remaining by shrinking the total number of ballots. We will show that the portion of first choice votes belonging to a candidate can at most double going into the next round. Note that this is different from before, as we are looking at the portion relative to the remaining votes in a round instead of the absolute number of votes. For $m = 2$ this is clearly the case, as the candidate who is not eliminated already must have at least half of the first choice vote share, so it can at most double. Assume $m > 2$. If one candidate holds all of the first choice votes, then the election is trivial and they clearly cannot gain or lose any votes, so assume one candidate does not hold all of the first choice votes. Let a be the minimum portion of first choice votes of the candidates remaining in a round. This is the portion of votes of the candidate eliminated. Now let b be the portion of votes of another candidate. Because no candidate has all of the votes and a is one of the minimum portions, $a + b < 1$. In order to maximize the increase in the vote share of the candidate with the portion b, assume all votes from the candidate eliminated either go into b or are removed from the election (i.e. they have no further rankings). Then let $a - x$ be the part of the portion given to b and x the part of the portion removed, with x ranging from 0 to a. The new portion of the candidate with the portion b is then $\frac{b+(a-x)}{1-x}$. The derivative with respect to x is $\frac{a+b-1}{(1-x)^2}$. This is negative in the domain of x and thus maximized when $x = 0$. Thus we need only consider the case when no votes are removed anyway, which was already shown in the main paper.

B Empirical Results

B.1 Description of Datasets

Our datasets are summarized in Tables 3 and 4.

Dataset	Profiles	Min, Max, Average Alternatives	Min, Max, Average Votes
APA Leader	12	5, 5, 5.0	13318, 20239, 16991.3
Debian Leader & Logo	8	4, 9, 6.3	143, 504, 419.0
Glasgow City Council	21	8, 13, 9.9	5199, 12744, 8970.3

Table 3: Preflib dataset descriptions.

Dataset	Profiles	Min, Max, Average Alternatives	Min, Max, Average Votes
Alaska House of Reps.	34	2, 4, 2.5	2119, 9816, 6319.8
Alaska Senate	18	2, 3, 2.4	6911, 16710, 12477.7
Berkeley City Council	24	2, 4, 3.0	1505, 10017, 6071.6
Minneapolis City Council	41	2, 8, 4.1	1928, 16457, 7225.4
Minneapolis Park Board	18	2, 5, 2.9	4697, 24466, 12705.7
NYC Democratic Council	46	2, 15, 6.4	6839, 39401, 17253.2
Oakland City Council	23	2, 7, 4.0	11182, 36890, 20115.0
Oakland School Director	18	2, 5, 2.9	10389, 34482, 19217.8
SanLeandro County Council	12	2, 4, 2.8	14203, 26953, 21242.4

Table 4: Harvard dataset descriptions.

B.2 Multiplicative Weights

The Multiplicative Weights Algorithm is a method to predict the outcome of an event using an ensemble of experts, which in our case are voting rules. The algorithm is initially equally likely to pick any of the experts to make its decision. The probabilities, or weights, associated with each expert get adjusted multiplicatively based on observing the outcomes of a series of training examples. The expected error of the ensemble is only slightly worse than that of the best expert in hindsight [Arora *et al.*, 2012].

For each statistical culture and sample size, we use 100 profiles and 100 random samples per profile as training data, for a total of 10000 iterations. Our learning rate, ϵ, is set to 0.001; with this learning rate, a rule that surpasses all others by several percentage points in accuracy will have a weight that dominates the others in the multiplicative weights ensemble, while the rules with the lowest accuracies may become very unlikely to be chosen at all.

Our pseudocode is presented in Algorithm 1. As in the body of the paper, all the learned multiplicative weights have been normalized between 0 and 1 in the synthetic results tables for clarity.

Algorithm 1 Multiplicative Weights Train

function MULTIPLICATIVE-WEIGHTS-TRAIN(ϵ)
    Initialize w_i^1 to 1 for all $i \in \mathcal{R}$

    for all profiles σ and samples s of σ **do**
        Pick a rule x from distribution $\mathcal{D}^t = \{p_1^t, p_2^t, \ldots, p_{|\mathcal{R}|}^t\}$, where $p_x^t = w_x^t / \sum_k w_k^t$
        for $y \in \mathcal{R}$ **do**
            if $RCV(\sigma) = x(s)$ and $x(s) = y(s)$ **then**
                $w_y^{t+1} \leftarrow w_y^t(1+\epsilon)$
            else if $RCV(\sigma) \neq x(s)$ and $x(s) \neq y(s)$ **then**
                $w_y^{t+1} \leftarrow w_y^t$
            else
                $w_y^{t+1} \leftarrow w_y^t(1-\epsilon)$
            end if
        end for
    end for
end function

B.3 Synthetic Results

We present our full results for different sample sizes (5, 10, 20, 30, 40, 50, 60, 70, 80, 90, and 100 percent) on synthetic data in Tables 1, 2, 6 to 9 and 11 to 15.

B.4 Real-World Results

We also present our results on real-world data from Preflib and the Harvard Dataverse in Figures 3 to 6.

		RCV	Plurality	Borda	Harm.	Cope.	MM	Bucklin	Pl. Veto	Veto
M, $\phi = 0.001$	ACC	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	25.37
	MPW	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	3.63e-8
M, $\phi = 0.01$	ACC	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	25.93
	MPW	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	3.65e-8
M, $\phi = 0.05$	ACC	99.95	99.87	99.81	99.77	99.93	99.93	99.84	**100.00**	27.90
	MPW	1.25e-1	1.25e-1	1.24e-1	1.23e-1	1.25e-1	1.25e-1	1.25e-1	1.27e-1	6.32e-8
M, $\phi = 0.1$	ACC	99.30	99.41	99.06	99.01	99.59	99.36	99.40	**99.98**	28.52
	MPW	1.28e-1	1.25e-1	1.18e-1	1.15e-1	1.28e-1	1.28e-1	1.25e-1	1.34e-1	1.16e-7
M, $\phi = 0.25$	ACC	93.37	92.04	91.43	89.77	93.14	93.68	92.16	**99.92**	33.60
	MPW	1.25e-1	9.30e-2	8.02e-2	6.84e-2	1.27e-1	1.28e-1	9.55e-2	2.82e-1	1.10e-6
M, $\phi = 0.5$	ACC	73.60	68.97	72.23	70.37	74.56	74.43	70.10	**97.46**	38.63
	MPW	2.36e-2	1.05e-2	1.50e-2	1.07e-2	2.74e-2	2.67e-2	1.03e-2	8.76e-1	2.59e-5
M, $\phi = 0.75$	ACC	44.15	41.44	46.70	43.86	45.96	44.69	43.30	**78.18**	33.49
	MPW	6.83e-3	4.26e-3	8.60e-3	5.88e-3	7.46e-3	7.75e-3	5.26e-3	9.53e-1	9.98e-4
M, $\phi = 0.95$	ACC	28.40	27.97	28.79	28.02	28.09	28.78	27.33	**39.81**	24.30
	MPW	9.29e-2	9.64e-2	8.72e-2	8.54e-2	8.24e-2	8.72e-2	7.96e-2	3.05e-1	8.39e-2
M, $\phi = 0.99$	ACC	26.96	26.13	**27.42**	26.15	26.10	27.17	25.98	24.00	22.51
	MPW	1.21e-1	1.11e-1	1.10e-1	1.10e-1	1.10e-1	1.10e-1	1.12e-1	9.93e-2	1.17e-1
M, $\phi = 0.999$	ACC	27.43	27.45	27.63	**28.41**	27.11	27.84	27.51	25.33	24.27
	MPW	1.09e-1	1.13e-1	1.17e-1	1.02e-1	1.10e-1	1.09e-1	1.20e-1	9.72e-2	1.22e-1
IC	ACC	**27.29**	26.73	26.58	25.98	26.59	27.00	25.48	25.58	23.33
	MPW	1.06e-1	1.20e-1	1.05e-1	1.07e-1	9.65e-2	1.00e-1	1.23e-1	1.21e-1	1.21e-1
Urn, $\alpha = 0.01$	ACC	29.69	29.72	30.59	30.02	**31.10**	30.66	30.41	26.03	24.40
	MPW	1.12e-1	1.13e-1	1.24e-1	1.19e-1	1.16e-1	1.16e-1	1.23e-1	9.08e-2	8.68e-2
Urn, $\alpha = 0.02$	ACC	31.58	30.14	**32.39**	31.16	31.66	32.25	30.47	28.86	25.72
	MPW	1.17e-1	1.11e-1	1.24e-1	1.17e-1	1.19e-1	1.21e-1	1.17e-1	8.63e-2	8.65e-2
Urn, $\alpha = 0.05$	ACC	**40.13**	39.50	38.56	39.00	39.34	39.21	37.30	28.86	24.71
	MPW	1.48e-1	1.52e-1	1.12e-1	1.20e-1	1.33e-1	1.37e-1	1.24e-1	4.25e-2	3.13e-2
Urn, $\alpha = 0.1$	ACC	**49.11**	47.91	45.51	47.30	47.58	48.17	44.63	43.36	26.07
	MPW	1.65e-1	1.42e-1	1.02e-1	1.18e-1	1.37e-1	1.50e-1	1.18e-1	5.97e-2	8.38e-3
Urn, $\alpha = 0.2$	ACC	**60.59**	58.58	51.83	56.33	57.51	57.17	54.36	49.62	25.82
	MPW	1.99e-1	1.86e-1	6.55e-2	1.08e-1	1.55e-1	1.51e-1	1.00e-1	3.43e-2	1.01e-3
Urn, $\alpha = 0.5$	ACC	**73.38**	71.86	66.01	69.70	71.39	72.09	69.03	59.44	25.39
	MPW	2.08e-1	1.63e-1	5.25e-2	9.71e-2	1.64e-1	1.77e-1	1.25e-1	1.38e-2	6.54e-5
SC	ACC	60.27	55.87	63.54	57.89	61.63	**61.79**	57.18	36.95	36.16
	MPW	1.61e-1	6.93e-2	1.96e-1	8.74e-2	1.97e-1	1.97e-1	8.72e-2	2.74e-3	1.73e-3
Conitzer SP	ACC	32.03	26.31	**35.70**	30.76	34.44	34.79	30.22	12.21	33.17
	MPW	1.01e-1	7.51e-2	1.71e-1	9.91e-2	1.41e-1	1.41e-1	9.89e-2	3.79e-2	1.35e-1
Conitzer SPOC	ACC	**27.91**	27.18	27.74	27.81	27.50	26.75	26.67	25.18	23.25
	MPW	1.11e-1	1.22e-1	1.14e-1	1.17e-1	9.88e-2	1.02e-1	1.10e-1	1.13e-1	1.12e-1
Walsh SP	ACC	55.44	46.67	**65.73**	56.53	61.13	61.53	47.60	43.94	32.86
	MPW	8.40e-2	1.92e-2	3.55e-1	7.12e-2	2.13e-1	2.13e-1	1.83e-2	2.48e-2	1.48e-3
1D Interval	ACC	**42.03**	41.18	37.47	40.07	41.17	40.88	37.96	33.22	21.31
	MPW	1.66e-1	1.69e-1	9.48e-2	1.24e-1	1.31e-1	1.31e-1	1.07e-1	6.21e-2	1.58e-2
2D Square	ACC	42.43	38.47	**46.89**	41.57	43.64	44.68	40.50	33.58	31.21
	MPW	1.20e-1	7.50e-2	2.02e-1	1.05e-1	1.57e-1	1.62e-1	1.09e-1	3.59e-2	3.52e-2
3D Cube	ACC	47.64	41.84	56.38	48.67	51.72	**52.61**	47.35	35.60	37.60
	MPW	1.09e-1	4.06e-2	3.04e-1	9.79e-2	1.67e-1	1.67e-1	7.61e-2	2.03e-2	1.81e-2
5D Cube	ACC	49.65	42.73	**56.15**	48.34	51.90	51.48	48.91	37.79	39.49
	MPW	1.18e-1	4.53e-2	2.71e-1	1.00e-1	1.73e-1	1.62e-1	9.10e-2	2.30e-2	1.63e-2
10D Cube	ACC	51.44	48.25	**56.38**	52.21	54.94	52.71	49.91	33.00	38.93
	MPW	1.25e-1	6.88e-2	2.27e-1	1.16e-1	1.68e-1	1.69e-1	1.03e-1	9.85e-3	1.34e-2
20D Cube	ACC	47.81	45.07	**50.14**	46.46	48.42	48.75	45.97	35.18	35.77
	MPW	1.36e-1	9.28e-2	1.82e-1	1.22e-1	1.59e-1	1.54e-1	1.11e-1	2.41e-2	1.92e-2
2D Sphere	ACC	36.75	**39.16**	31.14	35.54	34.14	34.14	32.10	27.94	13.08
	MPW	1.47e-1	2.04e-1	8.14e-2	1.46e-1	1.04e-1	1.06e-1	1.11e-1	7.14e-2	2.95e-2
3D Sphere	ACC	32.67	**34.65**	28.12	31.45	30.47	30.97	29.96	28.84	14.93
	MPW	1.40e-1	1.76e-1	8.17e-2	1.20e-1	1.03e-1	1.02e-1	1.25e-1	1.03e-1	4.96e-2
5D Sphere	ACC	31.08	**32.19**	27.99	30.10	28.81	28.91	29.11	26.38	17.11
	MPW	1.31e-1	1.58e-1	8.70e-2	1.36e-1	9.61e-2	1.02e-1	1.15e-1	1.15e-1	6.10e-2

Table 5: Accuracy of various rules predicting the RCV winner along with normalized multiplicative weights for 5% sample sizes.

		RCV	Plurality	Borda	Harm.	Cope.	MM	Bucklin	Pl. Veto	Veto
M, $\phi = 0.001$	ACC	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	25.32
	MPW	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	3.95e-8
M, $\phi = 0.01$	ACC	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	26.02
	MPW	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	5.24e-8
M, $\phi = 0.05$	ACC	100.00	100.00	100.00	100.00	99.99	100.00	100.00	100.00	28.57
	MPW	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	9.75e-8
M, $\phi = 0.1$	ACC	99.99	99.96	99.91	99.92	99.91	99.95	99.92	**100.00**	31.23
	MPW	1.25e-1	1.25e-1	1.24e-1	1.25e-1	1.25e-1	1.25e-1	1.24e-1	1.26e-1	1.45e-7
M, $\phi = 0.25$	ACC	97.76	97.79	97.04	97.76	97.86	98.09	95.88	**100.00**	39.43
	MPW	1.30e-1	1.24e-1	1.13e-1	1.16e-1	1.32e-1	1.29e-1	8.53e-2	1.71e-1	1.41e-6
M, $\phi = 0.5$	ACC	82.95	80.24	81.95	83.40	84.26	83.38	75.07	**99.96**	50.27
	MPW	6.02e-2	3.81e-2	4.81e-2	4.45e-2	6.17e-2	6.93e-2	1.11e-2	6.67e-1	9.86e-5
M, $\phi = 0.75$	ACC	51.72	48.54	54.59	52.26	54.84	53.95	49.34	**88.82**	39.02
	MPW	5.06e-3	2.42e-3	8.51e-3	6.29e-3	7.16e-3	6.38e-3	3.44e-3	9.60e-1	5.58e-4
M, $\phi = 0.95$	ACC	31.79	31.33	32.35	33.29	33.89	32.64	31.24	**40.94**	27.13
	MPW	8.78e-2	9.19e-2	9.40e-2	8.42e-2	8.58e-2	8.63e-2	7.80e-2	3.24e-1	6.81e-2
M, $\phi = 0.99$	ACC	30.46	29.22	30.82	**30.85**	30.45	30.40	28.75	18.59	23.96
	MPW	1.16e-1	1.25e-1	1.14e-1	1.16e-1	1.11e-1	1.11e-1	1.05e-1	9.48e-2	1.07e-1
M, $\phi = 0.999$	ACC	30.26	29.19	31.26	**31.67**	31.20	30.18	28.59	19.98	26.08
	MPW	1.13e-1	1.29e-1	1.14e-1	1.16e-1	1.20e-1	1.24e-1	1.15e-1	8.26e-2	8.55e-2
IC	ACC	29.38	28.18	30.00	30.27	**30.43**	29.87	27.29	20.36	25.59
	MPW	1.18e-1	1.22e-1	1.11e-1	1.10e-1	1.13e-1	1.19e-1	1.09e-1	9.62e-2	1.02e-1
Urn, $\alpha = 0.01$	ACC	34.30	34.21	34.79	**35.19**	34.75	33.92	30.80	24.44	26.19
	MPW	1.26e-1	1.21e-1	1.27e-1	1.21e-1	1.31e-1	1.38e-1	1.19e-1	5.33e-2	6.35e-2
Urn, $\alpha = 0.02$	ACC	36.68	36.33	38.12	38.28	**38.35**	37.54	34.79	20.10	27.12
	MPW	1.34e-1	1.26e-1	1.25e-1	1.26e-1	1.29e-1	1.34e-1	1.05e-1	5.19e-2	6.82e-2
Urn, $\alpha = 0.05$	ACC	46.01	45.81	45.16	**48.25**	45.71	45.68	37.49	24.30	27.50
	MPW	1.51e-1	1.61e-1	1.41e-1	1.54e-1	1.41e-1	1.42e-1	7.81e-2	1.08e-2	2.02e-2
Urn, $\alpha = 0.1$	ACC	54.33	54.55	53.09	**56.99**	54.71	54.46	46.70	27.23	27.39
	MPW	1.74e-1	1.76e-1	1.27e-1	1.67e-1	1.30e-1	1.60e-1	5.56e-2	8.88e-3	2.73e-3
Urn, $\alpha = 0.2$	ACC	**64.66**	62.89	57.34	63.41	63.74	63.32	55.06	38.62	27.28
	MPW	2.18e-1	2.48e-1	7.12e-2	1.53e-1	1.14e-1	1.59e-1	3.29e-2	3.96e-3	4.14e-4
Urn, $\alpha = 0.5$	ACC	75.81	**76.95**	67.30	76.09	72.90	74.60	67.59	57.67	26.38
	MPW	2.11e-1	1.88e-1	5.55e-2	1.46e-1	1.30e-1	2.08e-1	6.08e-2	1.75e-3	2.44e-5
SC	ACC	67.62	61.80	68.21	66.20	**68.95**	68.11	60.13	23.20	40.04
	MPW	1.54e-1	8.21e-2	1.97e-1	1.78e-1	1.75e-1	1.67e-1	4.63e-2	1.89e-4	1.08e-3
Conitzer SP	ACC	34.17	26.52	**41.31**	32.99	37.11	36.19	36.06	3.85	33.78
	MPW	1.17e-1	7.70e-2	2.09e-1	1.26e-1	1.40e-1	1.35e-1	1.07e-1	1.95e-2	6.96e-2
Conitzer SPOC	ACC	29.68	30.03	29.70	**31.35**	29.13	29.34	27.47	23.31	24.41
	MPW	1.20e-1	1.40e-1	1.06e-1	1.24e-1	1.05e-1	1.16e-1	1.02e-1	1.07e-1	7.95e-2
Walsh SP	ACC	62.15	54.92	**77.85**	64.74	72.91	69.49	58.11	35.41	32.42
	MPW	3.55e-2	1.01e-2	4.52e-1	1.04e-1	2.18e-1	1.69e-1	1.04e-2	1.14e-3	1.63e-4
1D Interval	ACC	47.31	48.59	38.33	**49.28**	42.12	43.76	38.87	28.36	19.42
	MPW	2.25e-1	2.15e-1	6.79e-2	1.69e-1	1.03e-1	1.28e-1	5.95e-2	2.48e-2	7.80e-3
2D Square	ACC	53.76	44.56	**59.15**	54.71	57.14	55.61	50.45	31.05	35.81
	MPW	1.13e-1	5.15e-2	2.26e-1	1.47e-1	1.86e-1	1.62e-1	9.47e-2	6.64e-3	1.40e-2
3D Cube	ACC	53.79	45.01	**63.12**	54.66	58.10	58.36	53.82	28.64	44.38
	MPW	8.45e-2	2.13e-2	3.34e-1	1.32e-1	1.85e-1	1.60e-1	6.95e-2	1.81e-3	1.16e-2
5D Cube	ACC	59.39	54.75	**64.93**	61.61	62.90	61.80	57.74	34.72	50.56
	MPW	1.01e-1	3.16e-2	2.92e-1	1.27e-1	1.85e-1	1.67e-1	7.35e-2	1.69e-3	2.06e-2
10D Cube	ACC	56.15	51.38	**61.60**	58.19	59.06	58.47	55.13	34.75	49.30
	MPW	1.20e-1	5.18e-2	2.39e-1	1.44e-1	1.94e-1	1.61e-1	7.75e-2	3.65e-4	1.24e-2
20D Cube	ACC	56.73	53.52	**59.48**	57.42	57.71	57.54	52.41	28.11	44.50
	MPW	1.28e-1	8.41e-2	2.18e-1	1.35e-1	1.79e-1	1.62e-1	7.30e-2	2.37e-3	1.87e-2
2D Sphere	ACC	55.04	46.04	**63.64**	55.66	61.41	59.30	52.59	29.80	9.33
	MPW	1.81e-1	2.73e-1	7.82e-2	1.83e-1	8.17e-2	1.00e-1	5.82e-2	3.48e-2	1.04e-2
3D Sphere	ACC	56.33	49.79	**63.36**	58.23	60.74	58.95	53.53	25.27	10.00
	MPW	1.68e-1	2.48e-1	7.76e-2	1.45e-1	8.63e-2	1.01e-1	8.31e-2	6.86e-2	2.30e-2
5D Sphere	ACC	59.14	52.34	**66.22**	61.59	63.30	62.03	59.53	38.00	15.04
	MPW	1.51e-1	2.14e-1	8.60e-2	1.36e-1	9.69e-2	1.04e-1	9.07e-2	9.09e-2	3.03e-2

Table 6: Accuracy of various rules predicting the RCV winner along with normalized multiplicative weights for 10% sample sizes.

		RCV	Plurality	Borda	Harm.	Cope.	MM	Bucklin	Pl. Veto	Veto
M, $\phi = 0.001$	ACC	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	25.50
	MPW	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	4.73e-8
M, $\phi = 0.01$	ACC	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	26.14
	MPW	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	5.56e-8
M, $\phi = 0.05$	ACC	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	30.36
	MPW	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.35e-7
M, $\phi = 0.1$	ACC	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	35.27
	MPW	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	3.20e-7
M, $\phi = 0.25$	ACC	99.79	99.80	99.53	99.86	99.84	99.84	99.37	**100.00**	47.45
	MPW	1.28e-1	1.27e-1	1.23e-1	1.24e-1	1.27e-1	1.27e-1	1.12e-1	1.32e-1	3.96e-6
M, $\phi = 0.5$	ACC	93.43	90.88	90.85	92.57	93.20	93.08	78.08	**100.00**	61.65
	MPW	1.31e-1	8.30e-2	9.59e-2	9.64e-2	1.34e-1	1.38e-1	6.91e-3	3.13e-1	2.56e-4
M, $\phi = 0.75$	ACC	63.89	58.86	64.71	64.48	66.12	65.47	57.71	**87.97**	49.05
	MPW	3.90e-2	1.09e-2	4.18e-2	2.05e-2	4.38e-2	4.76e-2	8.83e-3	7.86e-1	1.35e-3
M, $\phi = 0.95$	ACC	37.71	37.65	40.02	39.89	39.95	39.51	33.72	**43.23**	29.88
	MPW	1.12e-1	1.11e-1	1.19e-1	1.12e-1	1.05e-1	1.08e-1	6.93e-2	2.10e-1	5.34e-2
M, $\phi = 0.99$	ACC	35.79	33.25	36.16	35.86	**36.50**	36.23	31.51	16.61	26.46
	MPW	1.34e-1	1.33e-1	1.25e-1	1.37e-1	1.24e-1	1.33e-1	1.01e-1	4.40e-2	6.89e-2
M, $\phi = 0.999$	ACC	36.15	35.63	36.97	**37.04**	36.67	36.55	30.30	18.63	28.49
	MPW	1.33e-1	1.09e-1	1.37e-1	1.33e-1	1.34e-1	1.46e-1	1.06e-1	3.91e-2	6.11e-2
IC	ACC	34.27	32.77	35.32	34.53	**35.47**	33.78	30.13	19.43	28.95
	MPW	1.43e-1	1.23e-1	1.27e-1	1.25e-1	1.24e-1	1.35e-1	9.47e-2	5.07e-2	7.85e-2
Urn, $\alpha = 0.01$	ACC	40.79	40.18	41.01	**42.17**	40.98	41.48	33.70	20.76	30.14
	MPW	1.61e-1	1.09e-1	1.54e-1	1.38e-1	1.40e-1	1.64e-1	7.98e-2	1.57e-2	3.80e-2
Urn, $\alpha = 0.02$	ACC	45.33	41.27	46.01	**46.18**	45.76	45.45	38.55	16.25	32.45
	MPW	1.76e-1	1.10e-1	1.46e-1	1.35e-1	1.41e-1	1.70e-1	6.59e-2	1.42e-2	4.27e-2
Urn, $\alpha = 0.05$	ACC	55.60	53.89	52.38	**56.62**	54.60	54.82	41.06	19.64	31.65
	MPW	1.76e-1	1.54e-1	1.29e-1	1.81e-1	1.43e-1	1.76e-1	3.43e-2	8.90e-4	5.28e-3
Urn, $\alpha = 0.1$	ACC	65.55	61.50	59.36	**65.60**	63.99	63.12	50.59	20.18	29.97
	MPW	2.25e-1	1.54e-1	8.48e-2	1.95e-1	1.27e-1	1.85e-1	2.76e-2	1.32e-3	7.87e-4
Urn, $\alpha = 0.2$	ACC	**73.02**	68.07	63.45	70.68	71.48	71.86	59.48	30.77	28.83
	MPW	2.59e-1	2.16e-1	4.89e-2	2.00e-1	1.06e-1	1.54e-1	1.54e-2	3.60e-4	1.08e-4
Urn, $\alpha = 0.5$	ACC	81.05	**81.65**	71.68	80.20	76.85	79.32	71.53	51.21	28.58
	MPW	2.61e-1	1.70e-1	3.25e-2	1.58e-1	1.30e-1	2.04e-1	4.28e-2	1.67e-4	7.51e-6
SC	ACC	**76.02**	69.84	71.71	75.64	74.70	74.77	64.24	18.16	39.57
	MPW	2.38e-1	8.84e-2	9.78e-2	2.36e-1	1.55e-1	1.54e-1	3.14e-2	1.72e-5	2.98e-4
Conitzer SP	ACC	**41.24**	27.63	38.97	39.79	38.26	38.33	41.00	0.34	33.26
	MPW	2.21e-1	9.17e-2	1.41e-1	1.76e-1	9.76e-2	1.02e-1	1.11e-1	1.04e-2	4.86e-2
Conitzer SPOC	ACC	35.60	35.27	34.57	**36.98**	33.73	32.78	28.61	22.46	26.51
	MPW	1.63e-1	1.44e-1	1.24e-1	1.37e-1	1.15e-1	1.13e-1	8.51e-2	5.31e-2	6.59e-2
Walsh SP	ACC	71.73	64.37	**84.40**	75.40	82.12	81.61	57.74	21.05	33.31
	MPW	2.89e-2	6.28e-3	3.57e-1	9.62e-2	2.58e-1	2.52e-1	1.43e-3	8.68e-6	1.52e-5
1D Interval	ACC	55.44	53.67	40.50	**57.63**	46.85	46.99	42.04	24.90	20.88
	MPW	3.14e-1	1.89e-1	2.78e-2	2.75e-1	7.54e-2	8.07e-2	3.30e-2	3.03e-3	1.67e-3
2D Square	ACC	63.04	49.49	**66.14**	63.12	65.16	64.30	54.53	26.24	36.55
	MPW	2.04e-1	5.14e-2	1.40e-1	2.20e-1	1.54e-1	1.59e-1	6.55e-2	1.06e-3	5.07e-3
3D Cube	ACC	64.54	50.13	**69.04**	63.25	66.23	66.26	60.86	27.63	51.43
	MPW	1.26e-1	1.85e-2	2.66e-1	1.31e-1	2.08e-1	2.00e-1	4.46e-2	1.40e-4	5.02e-3
5D Cube	ACC	70.57	62.96	71.62	69.45	71.45	**71.65**	63.18	28.87	59.42
	MPW	1.16e-1	2.03e-2	3.05e-1	1.26e-1	1.78e-1	1.86e-1	4.86e-2	1.12e-4	1.98e-2
10D Cube	ACC	67.36	60.13	**71.26**	66.37	70.07	68.41	61.92	28.28	58.96
	MPW	1.39e-1	4.41e-2	2.46e-1	1.46e-1	1.79e-1	1.91e-1	4.42e-2	2.11e-5	1.10e-2
20D Cube	ACC	66.62	63.81	**69.20**	66.94	68.35	68.30	60.85	20.93	53.67
	MPW	1.53e-1	7.10e-2	2.29e-1	1.38e-1	1.70e-1	1.83e-1	4.34e-2	8.65e-5	1.27e-2
2D Sphere	ACC	66.46	52.03	68.12	67.18	**68.33**	68.02	57.40	23.16	7.37
	MPW	1.93e-1	3.18e-1	6.17e-2	2.38e-1	7.29e-2	8.46e-2	2.20e-2	7.47e-3	2.50e-3
3D Sphere	ACC	67.42	57.57	**70.64**	68.99	68.44	68.28	57.80	19.34	7.97
	MPW	1.99e-1	2.73e-1	8.09e-2	1.75e-1	8.17e-2	1.01e-1	5.93e-2	2.32e-2	6.80e-3
5D Sphere	ACC	70.37	62.07	**73.77**	70.96	72.24	71.63	65.85	29.13	13.56
	MPW	1.80e-1	2.81e-1	7.53e-2	1.66e-1	9.27e-2	9.92e-2	6.57e-2	2.79e-2	1.24e-2

Table 7: Accuracy of various rules predicting the RCV winner along with normalized multiplicative weights for 20% sample sizes.

		RCV	Plurality	Borda	Harm.	Cope.	MM	Bucklin	Pl. Veto	Veto
M, $\phi = 0.001$	ACC	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	24.42
	MPW	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	4.32e-8
M, $\phi = 0.01$	ACC	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	26.88
	MPW	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	5.08e-8
M, $\phi = 0.05$	ACC	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	32.87
	MPW	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	2.14e-7
M, $\phi = 0.1$	ACC	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	38.31
	MPW	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	4.79e-7
M, $\phi = 0.25$	ACC	99.95	99.97	99.95	99.97	99.97	99.98	99.92	**100.00**	53.67
	MPW	1.25e-1	1.24e-1	1.25e-1	1.25e-1	1.25e-1	1.26e-1	1.24e-1	1.26e-1	1.23e-5
M, $\phi = 0.5$	ACC	96.81	95.27	95.77	96.25	96.93	97.19	78.98	**100.00**	68.42
	MPW	1.55e-1	1.06e-1	1.13e-1	1.13e-1	1.54e-1	1.51e-1	4.50e-3	2.04e-1	4.20e-4
M, $\phi = 0.75$	ACC	72.25	65.53	72.15	71.45	73.66	73.36	62.87	**88.00**	53.96
	MPW	7.77e-2	1.93e-2	6.92e-2	3.81e-2	9.03e-2	9.20e-2	1.07e-2	6.01e-1	2.13e-3
M, $\phi = 0.95$	ACC	43.99	42.08	**46.00**	44.43	45.10	44.30	35.18	43.00	32.54
	MPW	1.40e-1	1.04e-1	1.43e-1	1.06e-1	1.36e-1	1.41e-1	5.66e-2	1.20e-1	5.29e-2
M, $\phi = 0.99$	ACC	39.99	37.08	41.01	41.14	**41.60**	40.38	33.14	17.05	27.52
	MPW	1.60e-1	1.41e-1	1.31e-1	1.38e-1	1.23e-1	1.39e-1	8.32e-2	3.28e-2	5.14e-2
M, $\phi = 0.999$	ACC	41.34	38.13	40.88	40.02	**41.26**	40.45	31.28	17.06	30.20
	MPW	1.47e-1	1.17e-1	1.70e-1	1.41e-1	1.54e-1	1.41e-1	6.67e-2	1.42e-2	4.87e-2
IC	ACC	39.67	36.50	39.62	38.91	**39.63**	39.39	31.23	18.02	30.05
	MPW	1.53e-1	1.19e-1	1.40e-1	1.30e-1	1.38e-1	1.41e-1	6.90e-2	3.83e-2	7.08e-2
Urn, $\alpha = 0.01$	ACC	**47.23**	42.90	45.09	45.88	45.31	45.19	35.23	21.06	31.54
	MPW	1.96e-1	8.59e-2	1.55e-1	1.47e-1	1.59e-1	1.69e-1	6.37e-2	6.22e-3	1.85e-2
Urn, $\alpha = 0.02$	ACC	50.96	46.98	50.93	50.97	50.98	**51.50**	42.11	17.31	35.23
	MPW	1.80e-1	9.08e-2	1.68e-1	1.34e-1	1.68e-1	1.68e-1	6.07e-2	5.52e-3	2.51e-2
Urn, $\alpha = 0.05$	ACC	61.30	59.38	58.88	**63.42**	60.46	58.89	43.99	18.93	32.12
	MPW	2.28e-1	1.33e-1	1.06e-1	2.00e-1	1.39e-1	1.70e-1	2.11e-2	2.60e-4	2.41e-3
Urn, $\alpha = 0.1$	ACC	**72.05**	66.44	62.41	70.23	67.64	67.76	52.49	18.43	30.26
	MPW	2.46e-1	1.36e-1	8.39e-2	2.27e-1	1.20e-1	1.74e-1	1.32e-2	1.75e-4	2.61e-4
Urn, $\alpha = 0.2$	ACC	**79.34**	69.69	65.55	73.46	75.28	75.65	62.21	26.80	28.48
	MPW	2.92e-1	2.25e-1	3.17e-2	2.10e-1	8.61e-2	1.47e-1	8.28e-3	9.96e-5	4.66e-5
Urn, $\alpha = 0.5$	ACC	**84.57**	83.30	72.37	83.11	79.08	81.38	73.40	49.20	28.35
	MPW	2.60e-1	1.76e-1	2.98e-2	1.73e-1	1.29e-1	1.95e-1	3.68e-2	4.51e-5	4.81e-6
SC	ACC	**81.22**	73.67	74.89	80.08	77.80	78.33	64.64	16.55	40.20
	MPW	2.82e-1	8.89e-2	7.32e-2	2.51e-1	1.40e-1	1.47e-1	1.85e-2	5.19e-6	1.87e-4
Conitzer SP	ACC	**45.15**	29.25	36.88	43.75	40.28	38.96	41.60	0.03	33.28
	MPW	3.16e-1	8.33e-2	8.43e-2	2.55e-1	6.89e-2	6.86e-2	8.76e-2	5.40e-3	3.16e-2
Conitzer SPOC	ACC	39.31	39.32	38.58	**40.17**	37.49	37.52	30.84	21.16	27.25
	MPW	1.68e-1	1.62e-1	1.34e-1	1.58e-1	1.18e-1	1.37e-1	5.76e-2	2.69e-2	3.90e-2
Walsh SP	ACC	77.05	71.41	85.66	81.87	**85.72**	85.19	55.99	11.87	33.14
	MPW	4.20e-2	1.22e-2	2.92e-1	1.42e-1	2.59e-1	2.52e-1	6.21e-4	5.57e-7	7.75e-6
1D Interval	ACC	62.02	55.54	40.11	**63.20**	48.22	47.52	43.76	24.23	19.96
	MPW	3.85e-1	1.71e-1	1.61e-2	3.10e-1	4.62e-2	4.72e-2	2.26e-2	1.67e-3	1.15e-3
2D Square	ACC	**69.83**	53.87	68.38	68.32	67.89	68.14	57.46	26.50	37.78
	MPW	2.48e-1	3.32e-2	1.08e-1	2.26e-1	1.63e-1	1.62e-1	5.64e-2	3.46e-4	3.28e-3
3D Cube	ACC	70.03	55.54	**71.71**	69.12	70.96	71.15	63.36	24.22	54.71
	MPW	1.67e-1	1.47e-2	2.17e-1	1.46e-1	2.06e-1	2.17e-1	2.80e-2	2.94e-5	4.97e-3
5D Cube	ACC	76.29	69.29	76.09	75.70	76.45	**76.85**	66.67	24.29	62.56
	MPW	1.64e-1	2.15e-2	2.44e-1	1.27e-1	2.00e-1	2.01e-1	2.73e-2	2.24e-5	1.53e-2
10D Cube	ACC	73.50	65.69	**76.26**	72.38	75.34	75.11	64.71	26.19	62.04
	MPW	1.62e-1	4.16e-2	2.25e-1	1.46e-1	1.83e-1	2.01e-1	3.20e-2	5.36e-6	9.70e-3
20D Cube	ACC	74.03	70.17	**75.32**	72.20	73.69	74.59	65.35	19.08	57.21
	MPW	1.75e-1	7.38e-2	2.00e-1	1.45e-1	1.77e-1	1.90e-1	3.08e-2	1.78e-5	9.69e-3
2D Sphere	ACC	73.13	56.06	69.81	71.81	**75.00**	73.47	59.91	20.33	6.01
	MPW	2.23e-1	2.96e-1	5.68e-2	2.54e-1	6.76e-2	8.28e-2	1.63e-2	2.49e-3	9.95e-4
3D Sphere	ACC	74.43	62.53	72.79	**73.87**	72.83	72.88	60.31	16.77	6.01
	MPW	2.23e-1	2.78e-1	6.55e-2	2.16e-1	7.24e-2	8.14e-2	4.94e-2	1.09e-2	3.62e-3
5D Sphere	ACC	74.65	68.09	77.11	76.68	76.67	**77.25**	68.61	28.91	12.55
	MPW	1.98e-1	2.76e-1	6.11e-2	2.26e-1	8.07e-2	9.45e-2	4.79e-2	1.07e-2	5.22e-3

Table 8: Accuracy of various rules predicting the RCV winner along with normalized multiplicative weights for 30% sample sizes.

		RCV	Plurality	Borda	Harm.	Cope.	MM	Bucklin	Pl. Veto	Veto
M, $\phi = 0.001$	ACC	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	25.15
	MPW	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	4.04e-8
M, $\phi = 0.01$	ACC	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	27.06
	MPW	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	7.09e-8
M, $\phi = 0.05$	ACC	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	33.77
	MPW	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	2.35e-7
M, $\phi = 0.1$	ACC	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	38.63
	MPW	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	8.02e-7
M, $\phi = 0.25$	ACC	100.00	100.00	100.00	100.00	99.99	100.00	99.99	100.00	58.30
	MPW	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.23e-1	1.25e-1	2.85e-5
M, $\phi = 0.5$	ACC	98.76	97.71	97.64	98.17	98.63	98.63	78.82	**100.00**	73.33
	MPW	1.48e-1	1.20e-1	1.24e-1	1.32e-1	1.50e-1	1.52e-1	3.52e-3	1.70e-1	9.11e-4
M, $\phi = 0.75$	ACC	77.92	70.48	77.15	75.86	78.59	78.02	66.65	**90.00**	58.15
	MPW	1.29e-1	2.18e-2	1.11e-1	5.17e-2	1.45e-1	1.51e-1	1.12e-2	3.76e-1	2.89e-3
M, $\phi = 0.95$	ACC	48.88	44.29	**50.43**	48.74	49.57	49.95	36.39	40.00	34.29
	MPW	1.46e-1	1.21e-1	1.55e-1	1.36e-1	1.47e-1	1.51e-1	3.74e-2	6.96e-2	3.67e-2
M, $\phi = 0.99$	ACC	44.42	39.52	**45.53**	43.99	45.01	45.05	32.38	15.01	29.15
	MPW	1.68e-1	1.45e-1	1.30e-1	1.63e-1	1.40e-1	1.40e-1	5.88e-2	1.39e-2	4.15e-2
M, $\phi = 0.999$	ACC	45.85	42.90	45.38	**46.20**	45.74	45.63	31.01	19.00	33.46
	MPW	1.66e-1	9.65e-2	1.75e-1	1.57e-1	1.55e-1	1.63e-1	4.20e-2	8.29e-3	3.66e-2
IC	ACC	43.45	39.78	**44.02**	42.18	41.97	43.55	30.60	16.00	30.94
	MPW	1.72e-1	1.03e-1	1.75e-1	1.31e-1	1.51e-1	1.53e-1	4.43e-2	1.80e-2	5.18e-2
Urn, $\alpha = 0.01$	ACC	**52.30**	47.22	49.14	50.82	48.63	49.38	35.55	20.00	33.96
	MPW	1.95e-1	7.17e-2	1.52e-1	1.59e-1	1.69e-1	2.01e-1	3.64e-2	2.48e-3	1.35e-2
Urn, $\alpha = 0.02$	ACC	55.53	49.43	**56.49**	54.96	55.07	56.07	42.74	16.03	36.10
	MPW	2.35e-1	7.95e-2	1.54e-1	1.37e-1	1.46e-1	1.83e-1	4.18e-2	3.47e-3	2.01e-2
Urn, $\alpha = 0.05$	ACC	**67.00**	61.49	62.51	66.19	64.27	63.93	45.69	16.38	33.66
	MPW	2.32e-1	9.98e-2	1.09e-1	1.93e-1	1.53e-1	1.98e-1	1.36e-2	9.43e-5	1.28e-3
Urn, $\alpha = 0.1$	ACC	**77.44**	68.41	64.72	72.89	69.94	72.85	54.62	18.19	30.85
	MPW	2.96e-1	1.15e-1	6.38e-2	1.97e-1	1.27e-1	1.91e-1	9.55e-3	8.83e-5	1.40e-4
Urn, $\alpha = 0.2$	ACC	**82.93**	72.66	68.62	75.88	77.85	78.50	63.20	24.44	28.66
	MPW	3.39e-1	1.86e-1	3.02e-2	2.05e-1	9.16e-2	1.42e-1	5.45e-3	3.36e-5	3.86e-5
Urn, $\alpha = 0.5$	ACC	**86.79**	84.27	73.71	83.04	80.58	82.80	74.74	46.78	28.56
	MPW	2.81e-1	1.54e-1	2.92e-2	1.75e-1	1.28e-1	1.99e-1	3.29e-2	1.94e-5	3.87e-6
SC	ACC	**85.62**	75.81	76.28	82.90	79.89	80.16	66.37	16.06	39.34
	MPW	3.49e-1	1.02e-1	5.00e-2	2.49e-1	1.18e-1	1.16e-1	1.64e-2	2.50e-6	8.98e-5
Conitzer SP	ACC	**50.09**	31.53	38.02	47.19	38.07	36.03	44.63	0.00	32.98
	MPW	3.47e-1	8.79e-2	6.24e-2	2.97e-1	4.75e-2	4.86e-2	8.92e-2	2.99e-3	1.75e-2
Conitzer SPOC	ACC	**44.79**	41.88	42.08	43.64	40.57	40.56	29.89	20.03	27.66
	MPW	1.59e-1	1.57e-1	1.49e-1	1.78e-1	1.37e-1	1.45e-1	3.78e-2	1.19e-2	2.57e-2
Walsh SP	ACC	81.16	74.79	**86.80**	83.68	86.38	86.63	55.02	6.32	34.34
	MPW	4.95e-2	1.61e-2	2.64e-1	1.67e-1	2.53e-1	2.50e-1	2.50e-4	8.78e-8	4.48e-6
1D Interval	ACC	**66.86**	58.41	40.88	65.66	48.60	48.12	43.98	22.05	19.21
	MPW	4.83e-1	1.70e-1	5.51e-3	2.81e-1	2.21e-2	2.44e-2	1.30e-2	8.01e-4	4.88e-4
2D Square	ACC	**72.72**	56.48	68.39	71.34	71.39	72.00	59.22	24.55	36.84
	MPW	3.07e-1	2.82e-2	8.46e-2	2.56e-1	1.35e-1	1.53e-1	3.53e-2	2.06e-4	1.59e-3
3D Cube	ACC	74.03	57.79	**74.77**	71.69	73.18	73.88	65.55	22.63	54.92
	MPW	1.76e-1	1.17e-2	1.66e-1	1.48e-1	2.29e-1	2.44e-1	2.28e-2	1.20e-5	2.31e-3
5D Cube	ACC	79.93	72.10	78.64	78.32	79.92	**80.19**	68.53	24.12	64.07
	MPW	1.92e-1	1.70e-2	2.15e-1	1.11e-1	2.05e-1	2.28e-1	2.34e-2	7.95e-6	8.92e-3
10D Cube	ACC	77.12	69.90	**80.04**	77.84	78.83	78.60	68.76	26.55	64.99
	MPW	1.66e-1	4.77e-2	2.20e-1	1.56e-1	1.93e-1	1.89e-1	2.18e-2	1.34e-6	6.24e-3
20D Cube	ACC	78.21	73.94	77.69	77.25	78.24	**78.76**	67.31	18.27	60.72
	MPW	1.87e-1	7.83e-2	1.59e-1	1.60e-1	1.93e-1	1.88e-1	2.62e-2	6.73e-6	7.75e-3
2D Sphere	ACC	**77.79**	57.23	72.12	77.22	76.08	76.98	64.05	18.86	4.71
	MPW	2.47e-1	2.65e-1	4.11e-2	3.23e-1	5.16e-2	6.31e-2	7.93e-3	1.22e-3	4.11e-4
3D Sphere	ACC	**78.40**	66.26	75.97	76.42	76.62	76.63	62.31	15.71	5.46
	MPW	2.45e-1	2.51e-1	4.91e-2	2.73e-1	6.86e-2	7.71e-2	3.08e-2	3.94e-3	1.12e-3
5D Sphere	ACC	78.68	72.77	79.52	80.55	**81.05**	80.93	71.53	25.68	11.45
	MPW	2.17e-1	2.93e-1	5.25e-2	2.72e-1	6.21e-2	7.21e-2	2.63e-2	4.30e-3	1.73e-3

Table 9: Accuracy of various rules predicting the RCV winner along with normalized multiplicative weights for 40% sample sizes.

		RCV	Plurality	Borda	Harm.	Cope.	MM	Bucklin	Pl. Veto	Veto
M, $\phi = 0.001$	ACC	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	25.53
	MPW	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	4.32e-8
M, $\phi = 0.01$	ACC	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	27.15
	MPW	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	7.19e-8
M, $\phi = 0.05$	ACC	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	34.34
	MPW	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	2.74e-7
M, $\phi = 0.1$	ACC	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	41.36
	MPW	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.07e-6
M, $\phi = 0.25$	ACC	100.00	100.00	100.00	100.00	100.00	100.00	99.98	100.00	61.99
	MPW	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	6.22e-5
M, $\phi = 0.5$	ACC	99.49	98.50	99.09	99.06	99.32	99.31	79.11	**100.00**	77.98
	MPW	1.47e-1	1.31e-1	1.34e-1	1.38e-1	1.46e-1	1.46e-1	3.53e-3	1.52e-1	1.75e-3
M, $\phi = 0.75$	ACC	82.30	75.00	79.64	79.41	83.01	82.33	69.85	**88.00**	62.25
	MPW	1.50e-1	2.15e-2	1.20e-1	6.38e-2	1.54e-1	1.73e-1	1.23e-2	3.02e-1	2.74e-3
M, $\phi = 0.95$	ACC	53.23	46.41	54.33	52.92	**54.83**	53.19	35.03	39.00	35.93
	MPW	1.69e-1	8.29e-2	1.89e-1	1.30e-1	1.79e-1	1.80e-1	1.75e-2	2.98e-2	2.33e-2
M, $\phi = 0.99$	ACC	**50.62**	44.24	49.06	48.23	49.32	50.13	32.17	19.00	30.41
	MPW	1.75e-1	1.48e-1	1.34e-1	1.69e-1	1.32e-1	1.76e-1	3.41e-2	7.12e-3	2.37e-2
M, $\phi = 0.999$	ACC	**51.22**	44.88	48.28	48.20	49.33	49.14	31.41	19.00	34.67
	MPW	2.02e-1	7.81e-2	1.92e-1	1.25e-1	1.74e-1	1.76e-1	2.68e-2	3.61e-3	2.23e-2
IC	ACC	47.27	41.08	**48.50**	46.42	46.53	45.36	30.49	20.01	32.09
	MPW	1.86e-1	7.78e-2	1.76e-1	1.17e-1	1.84e-1	1.73e-1	2.82e-2	9.85e-3	4.77e-2
Urn, $\alpha = 0.01$	ACC	**56.98**	50.23	51.11	53.24	52.71	53.22	35.71	21.00	33.18
	MPW	2.25e-1	5.60e-2	1.63e-1	1.47e-1	1.72e-1	2.13e-1	1.73e-2	7.70e-4	6.12e-3
Urn, $\alpha = 0.02$	ACC	**61.98**	49.98	57.87	58.19	59.08	59.51	44.23	18.01	37.98
	MPW	2.85e-1	7.38e-2	1.22e-1	1.46e-1	1.63e-1	1.75e-1	2.44e-2	1.41e-3	1.01e-2
Urn, $\alpha = 0.05$	ACC	**70.85**	64.05	63.02	69.64	66.65	69.04	45.57	16.07	33.42
	MPW	2.73e-1	7.33e-2	1.03e-1	1.65e-1	1.67e-1	2.09e-1	9.43e-3	4.88e-5	7.96e-4
Urn, $\alpha = 0.1$	ACC	**80.61**	70.68	65.66	74.14	72.46	75.33	54.97	16.57	31.27
	MPW	3.51e-1	9.65e-2	5.19e-2	1.94e-1	1.15e-1	1.86e-1	6.41e-3	4.93e-5	9.30e-5
Urn, $\alpha = 0.2$	ACC	**86.30**	71.89	69.33	77.66	80.53	81.28	64.48	23.97	29.18
	MPW	4.02e-1	1.85e-1	2.15e-2	1.86e-1	7.93e-2	1.22e-1	4.11e-3	1.92e-5	2.28e-5
Urn, $\alpha = 0.5$	ACC	**88.66**	84.78	73.34	85.06	81.67	83.83	75.59	45.24	28.60
	MPW	2.98e-1	1.51e-1	2.73e-2	1.74e-1	1.26e-1	1.95e-1	2.88e-2	1.10e-5	3.11e-6
SC	ACC	**87.91**	76.61	76.71	84.31	81.98	81.83	67.15	17.05	40.29
	MPW	3.93e-1	8.45e-2	3.83e-2	2.49e-1	1.14e-1	1.10e-1	1.23e-2	1.31e-6	6.83e-5
Conitzer SP	ACC	**53.66**	31.41	37.29	49.68	35.71	36.65	46.61	0.00	33.47
	MPW	4.42e-1	9.36e-2	2.88e-2	2.81e-1	2.43e-2	2.50e-2	8.49e-2	2.33e-3	1.79e-2
SPOC Conitzer	ACC	**49.46**	44.92	44.08	47.31	43.11	42.64	29.48	19.00	29.92
	MPW	2.17e-1	1.49e-1	1.42e-1	1.92e-1	1.32e-1	1.21e-1	1.72e-2	8.06e-3	2.15e-2
Walsh SP	ACC	83.55	79.78	**88.01**	86.17	85.84	87.87	53.33	3.77	33.16
	MPW	7.62e-2	4.16e-2	2.22e-1	2.11e-1	2.24e-1	2.26e-1	2.78e-4	8.95e-8	6.89e-6
1D Interval	ACC	**70.45**	59.56	42.50	67.94	49.49	50.42	45.25	22.89	20.00
	MPW	5.50e-1	1.06e-1	5.36e-3	2.84e-1	2.29e-2	2.41e-2	7.55e-3	3.17e-4	3.01e-4
2D Square	ACC	**77.71**	57.94	69.83	74.97	73.21	72.80	60.76	24.12	36.43
	MPW	4.04e-1	2.47e-2	5.33e-2	2.29e-1	1.28e-1	1.34e-1	2.62e-2	1.08e-4	1.07e-3
3D Cube	ACC	**77.78**	60.42	74.82	74.48	75.16	77.57	66.05	23.22	53.99
	MPW	2.15e-1	1.14e-2	1.49e-1	1.49e-1	2.20e-1	2.34e-1	1.90e-2	5.40e-6	2.49e-3
5D Cube	ACC	**82.84**	73.42	79.89	80.33	82.23	82.70	69.52	24.61	64.32
	MPW	1.91e-1	1.67e-2	2.02e-1	1.15e-1	2.27e-1	2.26e-1	1.62e-2	3.79e-6	6.31e-3
10D Cube	ACC	81.59	70.58	81.38	79.28	81.14	**82.44**	70.36	24.15	65.95
	MPW	1.98e-1	4.57e-2	2.09e-1	1.33e-1	1.87e-1	2.03e-1	1.85e-2	8.20e-7	5.75e-3
20D Cube	ACC	81.43	76.90	80.46	81.07	**81.49**	81.33	70.20	17.06	61.66
	MPW	2.17e-1	7.34e-2	1.55e-1	1.56e-1	1.76e-1	1.96e-1	2.13e-2	2.95e-6	5.36e-3
2D Sphere	ACC	**81.41**	59.11	73.54	78.79	76.44	78.80	63.29	18.49	5.30
	MPW	2.92e-1	2.11e-1	4.70e-2	3.07e-1	5.75e-2	7.82e-2	6.64e-3	6.97e-4	2.06e-4
3D Sphere	ACC	**82.32**	68.96	76.27	80.33	78.10	78.04	63.36	14.29	5.00
	MPW	3.47e-1	1.86e-1	4.97e-2	2.50e-1	6.40e-2	7.48e-2	2.43e-2	2.42e-3	8.00e-4
5D Sphere	ACC	82.15	74.15	81.43	82.57	82.52	**82.95**	72.38	25.95	10.41
	MPW	2.40e-1	3.10e-1	4.11e-2	2.68e-1	5.67e-2	6.64e-2	1.35e-2	3.69e-3	1.30e-3

Table 10: Accuracy of various rules predicting the RCV winner along with normalized multiplicative weights for 50% sample sizes.

		RCV	Plurality	Borda	Harm.	Cope.	MM	Bucklin	Pl. Veto	Veto
M, $\phi = 0.001$	ACC	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	25.84
	MPW	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	4.37e-8
M, $\phi = 0.01$	ACC	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	28.65
	MPW	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	8.15e-8
M, $\phi = 0.05$	ACC	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	35.19
	MPW	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	2.72e-7
M, $\phi = 0.1$	ACC	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	42.38
	MPW	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.41e-6
M, $\phi = 0.25$	ACC	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	64.68
	MPW	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.12e-4
M, $\phi = 0.5$	ACC	99.84	99.36	99.58	99.61	99.63	99.73	79.18	**100.00**	79.99
	MPW	1.44e-1	1.40e-1	1.35e-1	1.40e-1	1.44e-1	1.44e-1	3.42e-3	1.46e-1	3.27e-3
M, $\phi = 0.75$	ACC	86.03	78.33	83.92	82.66	85.42	85.53	71.97	**90.00**	64.80
	MPW	1.92e-1	2.31e-2	1.20e-1	6.57e-2	1.92e-1	2.04e-1	1.14e-2	1.88e-1	2.76e-3
M, $\phi = 0.95$	ACC	**59.39**	50.74	59.17	56.84	59.38	59.25	35.43	40.00	38.68
	MPW	1.88e-1	6.61e-2	1.93e-1	1.30e-1	1.78e-1	1.95e-1	9.88e-3	2.10e-2	1.91e-2
M, $\phi = 0.99$	ACC	**54.65**	46.46	52.27	52.62	53.51	53.08	31.45	16.00	33.62
	MPW	2.14e-1	1.38e-1	1.40e-1	1.74e-1	1.40e-1	1.63e-1	1.56e-2	3.52e-3	1.33e-2
M, $\phi = 0.999$	ACC	**55.39**	47.32	53.62	54.33	53.35	52.80	29.63	18.00	37.73
	MPW	2.25e-1	6.12e-2	1.90e-1	1.33e-1	1.77e-1	1.86e-1	1.31e-2	1.84e-3	1.35e-2
IC	ACC	**51.52**	43.23	50.32	48.82	48.99	50.51	30.22	17.00	32.75
	MPW	2.31e-1	6.73e-2	1.67e-1	1.18e-1	1.76e-1	1.91e-1	1.40e-2	5.29e-3	2.96e-2
Urn, $\alpha = 0.01$	ACC	**62.05**	52.07	54.52	57.38	54.69	55.81	34.70	20.00	37.76
	MPW	2.78e-1	4.47e-2	1.36e-1	1.39e-1	1.75e-1	2.12e-1	1.06e-2	4.34e-4	4.08e-3
Urn, $\alpha = 0.02$	ACC	**66.38**	50.85	62.66	61.53	60.31	64.21	45.03	17.00	38.58
	MPW	3.27e-1	5.36e-2	1.38e-1	1.20e-1	1.51e-1	1.78e-1	2.14e-2	5.02e-4	1.06e-2
Urn, $\alpha = 0.05$	ACC	**74.78**	67.02	67.24	73.12	70.42	71.52	47.10	18.03	34.39
	MPW	3.11e-1	5.27e-2	8.05e-2	1.55e-1	1.65e-1	2.33e-1	3.36e-3	1.74e-5	3.96e-4
Urn, $\alpha = 0.1$	ACC	**84.04**	72.25	66.19	76.49	75.03	76.50	54.54	17.23	30.40
	MPW	3.51e-1	7.09e-2	5.30e-2	1.94e-1	1.19e-1	2.07e-1	4.50e-3	2.16e-5	5.86e-5
Urn, $\alpha = 0.2$	ACC	**89.21**	72.72	70.09	78.71	81.98	81.95	64.24	22.45	30.50
	MPW	4.44e-1	1.51e-1	2.35e-2	1.81e-1	7.45e-2	1.22e-1	4.21e-3	8.77e-6	1.79e-5
Urn, $\alpha = 0.5$	ACC	**90.43**	85.32	74.07	85.90	82.58	84.57	75.85	43.38	29.09
	MPW	3.20e-1	1.47e-1	2.67e-2	1.65e-1	1.27e-1	1.90e-1	2.53e-2	7.35e-6	2.84e-6
SC	ACC	**90.19**	79.53	75.55	85.69	82.88	82.80	67.44	15.01	40.00
	MPW	4.39e-1	7.90e-2	3.08e-2	2.47e-1	9.59e-2	9.80e-2	9.87e-3	9.64e-7	5.17e-5
Conitzer SP	ACC	**58.84**	32.35	35.91	52.46	36.87	36.45	47.76	0.00	33.57
	MPW	5.25e-1	6.51e-2	1.79e-2	2.80e-1	1.61e-2	1.64e-2	6.79e-2	9.93e-4	1.02e-2
Conitzer SPOC	ACC	**53.98**	49.51	48.22	53.13	46.97	46.30	27.87	19.00	31.14
	MPW	2.11e-1	1.47e-1	1.68e-1	2.03e-1	1.28e-1	1.17e-1	8.55e-3	3.13e-3	1.48e-2
Walsh SP	ACC	86.43	81.51	86.99	86.08	86.94	**87.98**	52.88	2.04	33.48
	MPW	7.82e-2	4.81e-2	2.20e-1	2.09e-1	2.22e-1	2.22e-1	1.53e-4	2.67e-8	3.40e-6
1D Interval	ACC	**74.90**	63.69	42.08	70.33	48.76	50.39	47.54	21.88	19.55
	MPW	6.36e-1	9.18e-2	2.26e-3	2.42e-1	1.13e-2	1.16e-2	4.56e-3	1.94e-4	1.71e-4
2D Square	ACC	**79.76**	60.38	70.57	77.19	74.72	73.58	61.17	23.70	37.25
	MPW	4.82e-1	1.43e-2	3.97e-2	1.99e-1	1.20e-1	1.24e-1	2.03e-2	6.09e-5	5.64e-4
3D Cube	ACC	**80.69**	60.51	75.27	77.88	78.56	78.19	67.28	22.08	54.80
	MPW	2.54e-1	9.15e-3	1.22e-1	1.34e-1	2.23e-1	2.41e-1	1.48e-2	3.12e-6	2.04e-3
5D Cube	ACC	**85.14**	75.14	81.20	80.99	84.87	84.47	70.32	23.30	64.43
	MPW	2.27e-1	1.29e-2	1.64e-1	1.04e-1	2.30e-1	2.43e-1	1.48e-2	2.18e-6	4.52e-3
10D Cube	ACC	**84.15**	74.11	83.08	81.89	84.07	84.02	72.15	27.71	66.59
	MPW	2.00e-1	5.67e-2	1.85e-1	1.50e-1	1.83e-1	2.06e-1	1.48e-2	3.78e-7	4.44e-3
20D Cube	ACC	**84.51**	78.53	81.14	82.03	82.95	83.43	71.81	17.05	62.43
	MPW	2.25e-1	7.38e-2	1.46e-1	1.44e-1	1.81e-1	2.01e-1	2.37e-2	1.70e-6	4.38e-3
2D Sphere	ACC	**84.60**	59.39	76.12	80.55	81.98	79.40	64.66	18.86	5.37
	MPW	3.74e-1	1.61e-1	4.10e-2	2.96e-1	5.74e-2	6.53e-2	3.81e-3	3.78e-4	1.06e-4
3D Sphere	ACC	**85.44**	69.77	77.73	81.28	80.15	80.25	64.58	15.13	5.94
	MPW	4.15e-1	1.46e-1	3.77e-2	2.49e-1	5.79e-2	7.38e-2	1.92e-2	1.27e-3	3.08e-4
5D Sphere	ACC	**84.95**	77.10	83.23	83.97	84.17	84.60	72.46	26.68	10.55
	MPW	3.14e-1	2.53e-1	3.34e-2	2.68e-1	5.35e-2	6.69e-2	8.82e-3	1.68e-3	6.52e-4

Table 11: Accuracy of various rules predicting the RCV winner along with normalized multiplicative weights for 60% sample sizes.

		RCV	Plurality	Borda	Harm.	Cope.	MM	Bucklin	Pl. Veto	Veto
M, $\phi = 0.001$	ACC MPW	100.00 1.25e-1	100.00 1.25e-1	100.00 1.25e-1	100.00 1.25e-1	100.00 1.25e-1	100.00 1.25e-1	100.00 1.25e-1	100.00 1.25e-1	25.41 5.27e-8
M, $\phi = 0.01$	ACC MPW	100.00 1.25e-1	100.00 1.25e-1	100.00 1.25e-1	100.00 1.25e-1	100.00 1.25e-1	100.00 1.25e-1	100.00 1.25e-1	100.00 1.25e-1	28.67 7.22e-8
M, $\phi = 0.05$	ACC MPW	100.00 1.25e-1	100.00 1.25e-1	100.00 1.25e-1	100.00 1.25e-1	100.00 1.25e-1	100.00 1.25e-1	100.00 1.25e-1	100.00 1.25e-1	36.25 2.89e-7
M, $\phi = 0.1$	ACC MPW	100.00 1.25e-1	100.00 1.25e-1	100.00 1.25e-1	100.00 1.25e-1	100.00 1.25e-1	100.00 1.25e-1	100.00 1.25e-1	100.00 1.25e-1	43.42 1.84e-6
M, $\phi = 0.25$	ACC MPW	100.00 1.25e-1	100.00 1.25e-1	100.00 1.25e-1	100.00 1.25e-1	100.00 1.25e-1	100.00 1.25e-1	100.00 1.25e-1	100.00 1.25e-1	66.63 1.56e-4
M, $\phi = 0.5$	ACC MPW	99.92 1.42e-1	99.64 1.41e-1	99.81 1.41e-1	99.85 1.41e-1	99.90 1.42e-1	99.96 1.42e-1	79.17 3.48e-3	**100.00** 1.42e-1	82.96 4.81e-3
M, $\phi = 0.75$	ACC MPW	88.74 2.13e-1	79.91 2.10e-2	85.43 1.23e-1	85.70 6.47e-2	87.62 2.11e-1	88.40 2.33e-1	74.35 9.49e-3	**89.00** 1.21e-1	68.01 3.84e-3
M, $\phi = 0.95$	ACC MPW	63.78 2.02e-1	52.72 4.37e-2	63.35 2.07e-1	62.11 1.12e-1	**64.51** 1.89e-1	63.03 2.26e-1	34.97 3.41e-3	42.00 8.49e-3	41.08 9.67e-3
M, $\phi = 0.99$	ACC MPW	**60.50** 2.45e-1	47.79 1.11e-1	57.31 1.42e-1	57.77 1.71e-1	58.62 1.46e-1	58.90 1.68e-1	29.63 6.84e-3	18.00 1.47e-3	31.59 8.91e-3
M, $\phi = 0.999$	ACC MPW	**61.94** 2.45e-1	50.89 4.28e-2	57.67 2.11e-1	57.54 1.25e-1	57.08 1.78e-1	57.90 1.83e-1	28.56 4.54e-3	21.00 6.79e-4	39.67 8.89e-3
IC	ACC MPW	**57.03** 2.42e-1	47.46 5.23e-2	53.29 1.97e-1	52.42 1.07e-1	55.50 1.79e-1	55.55 1.99e-1	29.27 4.51e-3	20.00 1.62e-3	35.76 1.79e-2
Urn, $\alpha = 0.01$	ACC MPW	**66.20** 2.73e-1	54.10 2.72e-2	57.03 1.38e-1	58.35 1.24e-1	60.61 1.78e-1	59.58 2.52e-1	36.03 6.03e-3	20.00 1.46e-4	36.29 1.74e-3
Urn, $\alpha = 0.02$	ACC MPW	**70.85** 4.17e-1	53.42 3.04e-2	64.06 1.32e-1	62.91 8.47e-2	66.69 1.48e-1	65.64 1.71e-1	43.71 1.08e-2	18.00 2.26e-4	39.64 6.42e-3
Urn, $\alpha = 0.05$	ACC MPW	**78.38** 3.72e-1	68.87 3.29e-2	68.44 6.78e-2	75.59 1.28e-1	72.69 1.70e-1	74.46 2.26e-1	47.50 2.95e-3	19.02 1.28e-5	35.58 2.29e-4
Urn, $\alpha = 0.1$	ACC MPW	**86.73** 4.63e-1	71.48 5.30e-2	66.67 4.19e-2	78.18 1.66e-1	74.13 1.05e-1	79.04 1.67e-1	55.33 3.81e-3	17.13 1.60e-5	30.55 3.95e-5
Urn, $\alpha = 0.2$	ACC MPW	**90.51** 4.62e-1	72.43 1.56e-1	73.39 1.56e-2	81.70 1.74e-1	82.98 6.88e-2	82.34 1.21e-1	64.65 2.73e-3	21.91 7.72e-6	30.50 1.33e-5
Urn, $\alpha = 0.5$	ACC MPW	**92.31** 3.39e-1	85.22 1.44e-1	74.39 2.62e-2	84.66 1.70e-1	83.10 1.21e-1	85.15 1.74e-1	76.69 2.55e-2	42.17 5.01e-6	29.75 2.37e-6
SC	ACC MPW	**91.77** 4.51e-1	81.53 8.39e-2	78.14 1.93e-2	86.29 2.94e-1	84.00 7.12e-2	83.85 7.27e-2	67.58 7.14e-3	15.00 6.42e-7	40.89 2.74e-5
Conitzer SP	ACC MPW	**62.73** 6.80e-1	34.40 4.59e-2	39.02 8.40e-3	55.98 2.12e-1	34.53 6.86e-3	35.98 6.80e-3	49.09 3.30e-2	0.00 4.28e-4	32.93 5.88e-3
Conitzer SPOC	ACC MPW	**58.88** 2.96e-1	53.81 1.41e-1	50.71 1.24e-1	57.17 2.14e-1	50.43 1.03e-1	49.93 1.10e-1	27.69 3.72e-3	20.00 2.01e-3	33.54 6.95e-3
Walsh SP	ACC MPW	**88.67** 1.21e-1	84.74 9.76e-2	88.00 1.91e-1	88.49 2.07e-1	86.99 1.91e-1	88.01 1.92e-1	51.70 1.87e-4	1.42 3.33e-8	33.30 4.87e-6
1D Interval	ACC MPW	**78.25** 7.37e-1	61.12 6.44e-2	40.65 1.33e-3	71.81 1.78e-1	49.75 8.03e-3	51.09 7.67e-3	46.64 3.60e-3	21.91 1.38e-4	19.35 1.08e-4
2D Square	ACC MPW	**83.12** 5.96e-1	60.51 1.30e-2	71.46 2.12e-2	78.48 1.77e-1	74.44 8.91e-2	74.76 9.15e-2	61.85 1.18e-2	24.43 3.31e-5	37.34 3.75e-4
3D Cube	ACC MPW	**82.93** 2.74e-1	63.64 8.36e-3	74.93 8.07e-2	77.07 1.01e-1	81.05 2.53e-1	78.91 2.68e-1	67.69 1.36e-2	22.02 1.39e-6	57.08 1.29e-3
5D Cube	ACC MPW	**87.58** 2.47e-1	76.80 1.26e-2	81.04 1.50e-1	84.78 9.99e-2	86.03 2.36e-1	86.38 2.39e-1	70.82 1.14e-2	23.17 1.51e-6	64.48 4.32e-3
10D Cube	ACC MPW	86.51 2.17e-1	75.87 4.87e-2	84.88 1.49e-1	83.91 1.39e-1	86.37 2.08e-1	**87.19** 2.23e-1	73.26 1.12e-2	25.44 2.75e-7	67.79 4.08e-3
20D Cube	ACC MPW	**86.29** 2.34e-1	83.31 7.56e-2	84.30 1.35e-1	84.20 1.61e-1	85.27 1.73e-1	85.16 1.98e-1	72.79 1.84e-2	18.02 9.89e-7	62.25 3.93e-3
2D Sphere	ACC MPW	**88.38** 4.78e-1	60.50 1.19e-1	73.34 3.32e-2	83.18 2.62e-1	81.70 4.59e-2	80.60 5.98e-2	64.77 2.15e-3	17.39 1.64e-4	5.07 4.21e-5
3D Sphere	ACC MPW	**87.85** 5.14e-1	71.35 1.18e-1	77.68 2.91e-2	83.00 2.23e-1	82.10 4.60e-2	81.58 5.94e-2	65.37 8.93e-3	14.05 6.37e-4	4.29 1.39e-4
5D Sphere	ACC MPW	87.36 4.75e-1	78.09 1.59e-1	83.64 2.27e-2	85.84 2.42e-1	87.06 4.47e-2	**87.68** 5.16e-2	74.80 3.97e-3	25.54 7.64e-4	11.98 2.36e-4

Table 12: Accuracy of various rules predicting the RCV winner along with normalized multiplicative weights for 70% sample sizes.

		RCV	Plurality	Borda	Harm.	Cope.	MM	Bucklin	Pl. Veto	Veto
M, $\phi = 0.001$	ACC	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	25.82
	MPW	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	4.10e-8
M, $\phi = 0.01$	ACC	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	29.09
	MPW	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	8.64e-8
M, $\phi = 0.05$	ACC	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	35.74
	MPW	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	3.21e-7
M, $\phi = 0.1$	ACC	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	44.85
	MPW	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	2.25e-6
M, $\phi = 0.25$	ACC	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	69.08
	MPW	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	2.48e-4
M, $\phi = 0.5$	ACC	100.00	99.81	99.97	99.95	99.97	100.00	79.23	100.00	84.73
	MPW	1.41e-1	1.41e-1	1.41e-1	1.41e-1	1.41e-1	1.41e-1	4.26e-3	1.41e-1	7.21e-3
M, $\phi = 0.75$	ACC	**91.90**	82.91	87.88	87.70	90.68	90.85	75.33	89.00	70.50
	MPW	2.21e-1	2.08e-2	1.34e-1	6.01e-2	2.24e-1	2.38e-1	7.56e-3	9.08e-2	4.46e-3
M, $\phi = 0.95$	ACC	**71.44**	56.51	67.26	66.69	68.19	68.03	33.05	39.00	41.50
	MPW	2.85e-1	3.03e-2	1.44e-1	1.05e-1	2.02e-1	2.20e-1	1.60e-3	5.79e-3	6.34e-3
M, $\phi = 0.99$	ACC	**66.38**	50.45	64.91	60.12	64.43	65.22	27.68	16.00	34.03
	MPW	3.21e-1	8.98e-2	1.19e-1	1.59e-1	1.26e-1	1.78e-1	2.55e-3	7.46e-4	3.26e-3
M, $\phi = 0.999$	ACC	**67.49**	53.67	61.70	62.79	63.82	63.32	27.44	17.00	39.34
	MPW	3.02e-1	2.12e-2	1.98e-1	9.01e-2	1.77e-1	2.04e-1	1.78e-3	2.01e-4	6.11e-3
IC	ACC	**62.04**	49.37	57.91	57.91	58.81	60.77	27.33	18.00	37.24
	MPW	2.89e-1	4.30e-2	1.73e-1	1.10e-1	1.67e-1	2.05e-1	1.52e-3	5.34e-4	1.11e-2
Urn, $\alpha = 0.01$	ACC	**71.61**	54.07	59.17	61.58	63.35	64.43	34.69	21.00	37.97
	MPW	4.13e-1	2.32e-2	8.21e-2	1.26e-1	1.35e-1	2.18e-1	1.89e-3	6.14e-5	7.94e-4
Urn, $\alpha = 0.02$	ACC	**75.74**	57.81	66.28	67.21	69.71	68.58	43.84	17.00	38.98
	MPW	5.62e-1	2.52e-2	6.82e-2	8.67e-2	1.23e-1	1.25e-1	5.42e-3	1.46e-4	3.50e-3
Urn, $\alpha = 0.05$	ACC	**82.79**	71.11	70.66	78.51	74.70	79.35	48.34	19.01	34.06
	MPW	4.57e-1	2.54e-2	4.98e-2	9.56e-2	1.66e-1	2.05e-1	1.52e-3	5.54e-6	1.23e-4
Urn, $\alpha = 0.1$	ACC	**88.82**	73.56	67.70	78.87	77.63	81.02	57.43	16.07	31.90
	MPW	5.18e-1	3.71e-2	3.62e-2	1.54e-1	9.93e-2	1.51e-1	2.96e-3	1.16e-5	2.68e-5
Urn, $\alpha = 0.2$	ACC	**93.52**	73.58	72.34	81.25	84.09	84.77	65.84	21.04	31.28
	MPW	5.13e-1	1.40e-1	1.55e-2	1.77e-1	5.61e-2	9.47e-2	2.38e-3	4.71e-6	1.20e-5
Urn, $\alpha = 0.5$	ACC	**94.23**	85.85	74.08	85.10	83.85	85.94	76.96	41.41	29.56
	MPW	3.71e-1	1.32e-1	2.52e-2	1.64e-1	1.18e-1	1.66e-1	2.33e-2	3.57e-6	1.92e-6
SC	ACC	**93.64**	82.23	80.70	85.58	84.76	84.70	68.50	16.00	39.88
	MPW	4.62e-1	7.16e-2	1.44e-2	3.04e-1	7.15e-2	7.01e-2	6.37e-3	4.63e-7	2.08e-5
Conitzer SP	ACC	**68.15**	33.52	32.55	57.74	33.54	35.01	50.70	0.00	33.54
	MPW	7.71e-1	3.84e-2	2.72e-3	1.61e-1	2.48e-3	2.44e-3	2.03e-2	1.23e-4	2.27e-3
Conitzer SPOC	ACC	**66.22**	55.31	56.28	61.01	53.23	56.41	26.47	20.00	33.98
	MPW	3.46e-1	1.18e-1	1.24e-1	2.17e-1	9.32e-2	9.51e-2	1.40e-3	6.06e-4	5.25e-3
Walsh SP	ACC	**90.35**	85.07	86.00	86.25	87.00	87.00	52.37	0.66	33.29
	MPW	1.58e-1	9.89e-2	1.84e-1	1.90e-1	1.84e-1	1.84e-1	1.21e-4	2.04e-8	2.95e-6
1D Interval	ACC	**82.04**	62.84	41.10	72.85	52.09	49.02	48.43	21.96	20.24
	MPW	8.52e-1	2.70e-2	5.77e-4	1.12e-1	3.53e-3	3.59e-3	1.41e-3	4.40e-5	3.79e-5
2D Square	ACC	**86.14**	61.96	70.89	78.66	77.35	76.34	62.50	22.18	36.64
	MPW	6.36e-1	8.57e-3	1.85e-2	1.44e-1	8.75e-2	9.50e-2	1.00e-2	1.20e-5	2.14e-4
3D Cube	ACC	**86.77**	62.49	75.76	80.45	82.09	82.44	67.25	23.01	57.86
	MPW	3.73e-1	1.03e-2	5.29e-2	1.11e-1	2.16e-1	2.27e-1	1.02e-2	1.29e-6	7.80e-4
5D Cube	ACC	**90.14**	76.61	85.05	85.79	88.38	88.48	69.54	24.02	64.29
	MPW	2.68e-1	9.72e-3	1.31e-1	8.77e-2	2.42e-1	2.49e-1	8.56e-3	9.95e-7	3.21e-3
10D Cube	ACC	**90.36**	79.72	84.18	84.95	89.19	89.39	74.70	23.46	68.98
	MPW	2.54e-1	4.93e-2	1.49e-1	1.23e-1	1.94e-1	2.17e-1	1.01e-2	1.54e-7	3.54e-3
20D Cube	ACC	**89.19**	82.41	85.96	86.26	86.61	87.64	74.01	18.00	63.94
	MPW	2.49e-1	8.66e-2	1.11e-1	1.71e-1	1.73e-1	1.93e-1	1.52e-2	6.67e-7	1.82e-3
2D Sphere	ACC	**90.81**	61.77	75.31	84.58	84.66	82.27	67.30	17.18	3.60
	MPW	6.59e-1	5.30e-2	2.90e-2	1.54e-1	4.67e-2	5.64e-2	1.20e-3	6.09e-5	1.20e-5
3D Sphere	ACC	**90.17**	75.13	78.88	83.93	83.84	83.80	66.67	14.04	3.89
	MPW	6.89e-1	5.87e-2	2.07e-2	1.31e-1	3.77e-2	5.78e-2	5.20e-3	3.23e-4	4.55e-5
5D Sphere	ACC	**90.27**	80.30	83.57	89.06	90.08	89.74	74.56	25.26	10.55
	MPW	5.91e-1	1.31e-1	1.35e-2	1.94e-1	2.82e-2	4.14e-2	1.40e-3	2.73e-4	8.16e-5

Table 13: Accuracy of various rules predicting the RCV winner along with normalized multiplicative weights for 80% sample sizes.

		RCV	Plurality	Borda	Harm.	Cope.	MM	Bucklin	Pl. Veto	Veto
M, $\phi = 0.001$	ACC	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	25.65
	MPW	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	4.89e-8
M, $\phi = 0.01$	ACC	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	29.92
	MPW	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.02e-7
M, $\phi = 0.05$	ACC	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	35.52
	MPW	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	3.79e-7
M, $\phi = 0.1$	ACC	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	45.40
	MPW	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	2.66e-6
M, $\phi = 0.25$	ACC	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	69.68
	MPW	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	3.15e-4
M, $\phi = 0.5$	ACC	100.00	99.98	100.00	99.99	100.00	100.00	80.08	100.00	86.56
	MPW	1.41e-1	1.41e-1	1.41e-1	1.41e-1	1.41e-1	1.41e-1	3.72e-3	1.41e-1	1.04e-2
M, $\phi = 0.75$	ACC	**94.75**	84.67	89.14	89.56	93.55	93.12	77.79	90.00	72.99
	MPW	2.29e-1	2.48e-2	1.24e-1	7.21e-2	2.22e-1	2.36e-1	7.31e-3	8.18e-2	3.20e-3
M, $\phi = 0.95$	ACC	**78.40**	61.07	73.02	69.27	74.74	75.54	32.39	41.00	44.23
	MPW	3.30e-1	1.23e-2	1.46e-1	6.79e-2	1.99e-1	2.39e-1	3.04e-4	1.66e-3	2.55e-3
M, $\phi = 0.99$	ACC	**74.64**	53.05	67.55	63.44	69.27	69.08	25.35	16.00	33.73
	MPW	4.70e-1	4.88e-2	7.70e-2	8.94e-2	1.23e-1	1.88e-1	7.42e-4	2.13e-4	2.06e-3
M, $\phi = 0.999$	ACC	**76.17**	56.26	66.35	65.52	69.43	69.92	25.96	15.00	42.91
	MPW	3.73e-1	1.56e-2	1.77e-1	7.88e-2	1.69e-1	1.83e-1	4.97e-4	6.50e-5	2.27e-3
IC	ACC	**69.35**	53.14	63.72	60.71	63.58	65.49	27.66	17.00	40.44
	MPW	3.94e-1	2.60e-2	1.61e-1	8.63e-2	1.33e-1	1.96e-1	2.43e-4	1.56e-4	2.75e-3
Urn, $\alpha = 0.01$	ACC	**78.26**	58.06	62.42	66.27	67.70	69.14	35.99	20.00	37.14
	MPW	5.35e-1	1.31e-2	6.32e-2	1.11e-1	1.17e-1	1.59e-1	6.65e-4	2.29e-5	4.31e-4
Urn, $\alpha = 0.02$	ACC	**82.61**	55.79	67.85	70.73	73.60	72.17	42.76	18.00	42.85
	MPW	6.61e-1	1.37e-2	4.79e-2	5.92e-2	1.01e-1	1.12e-1	3.46e-3	5.47e-5	1.88e-3
Urn, $\alpha = 0.05$	ACC	**87.85**	72.06	70.54	78.58	77.08	79.70	48.04	17.00	33.66
	MPW	5.92e-1	1.43e-2	2.95e-2	7.26e-2	1.37e-1	1.54e-1	8.28e-4	2.47e-6	4.71e-5
Urn, $\alpha = 0.1$	ACC	**91.89**	73.53	67.12	79.41	78.86	82.12	55.53	17.01	31.23
	MPW	5.82e-1	2.98e-2	2.72e-2	1.34e-1	1.08e-1	1.17e-1	2.32e-3	6.76e-6	1.85e-5
Urn, $\alpha = 0.2$	ACC	**95.72**	73.18	72.47	82.82	86.41	85.80	65.31	19.49	32.28
	MPW	5.67e-1	1.08e-1	1.52e-2	1.57e-1	4.76e-2	1.03e-1	2.21e-3	2.53e-6	1.15e-5
Urn, $\alpha = 0.5$	ACC	**95.76**	85.81	74.49	85.04	84.79	86.23	78.59	40.33	29.50
	MPW	4.09e-1	1.18e-1	2.25e-2	1.64e-1	1.11e-1	1.55e-1	1.99e-2	2.42e-6	1.63e-6
SC	ACC	**95.71**	83.85	78.83	87.34	85.66	85.48	68.31	16.00	39.81
	MPW	5.24e-1	5.02e-2	1.30e-2	2.59e-1	7.39e-2	7.37e-2	6.16e-3	3.40e-7	1.92e-5
Conitzer SP	ACC	**76.13**	33.91	38.06	59.57	35.17	35.44	52.58	0.00	33.24
	MPW	9.22e-1	8.64e-3	4.54e-4	5.44e-2	4.61e-4	4.53e-4	1.26e-2	1.95e-5	6.49e-4
Conitzer SPOC	ACC	**74.74**	61.12	57.47	65.89	59.43	60.33	25.25	20.00	37.21
	MPW	5.49e-1	6.69e-2	7.35e-2	1.95e-1	5.94e-2	5.35e-2	4.27e-4	2.07e-4	1.96e-3
Walsh SP	ACC	**93.43**	86.12	87.00	87.32	87.00	86.00	52.58	0.40	32.69
	MPW	2.14e-1	1.28e-1	1.62e-1	1.73e-1	1.62e-1	1.62e-1	1.01e-4	1.43e-8	2.93e-6
1D Interval	ACC	**87.87**	63.28	40.04	73.60	49.18	49.27	47.87	22.98	19.13
	MPW	9.28e-1	1.46e-2	2.12e-4	5.30e-2	1.64e-3	1.65e-3	7.10e-4	1.86e-5	1.80e-5
2D Square	ACC	**89.40**	63.96	73.00	77.69	76.67	76.52	64.46	22.11	37.03
	MPW	7.37e-1	3.70e-3	1.32e-2	9.93e-2	6.94e-2	7.23e-2	4.95e-3	4.63e-6	9.59e-5
3D Cube	ACC	**90.34**	64.70	77.99	81.82	82.48	82.57	68.24	22.00	59.56
	MPW	4.28e-1	7.50e-3	4.38e-2	9.10e-2	2.06e-1	2.16e-1	7.50e-3	6.99e-7	7.96e-4
5D Cube	ACC	**93.39**	77.55	86.60	86.89	89.76	90.33	72.45	24.00	64.65
	MPW	2.87e-1	9.03e-3	1.15e-1	8.97e-2	2.37e-1	2.53e-1	8.08e-3	6.31e-7	1.82e-3
10D Cube	ACC	**92.92**	79.66	87.42	88.08	90.11	92.08	74.98	26.22	68.78
	MPW	2.75e-1	4.31e-2	1.29e-1	1.17e-1	1.93e-1	2.33e-1	7.53e-3	8.68e-8	2.67e-3
20D Cube	ACC	**91.40**	84.38	86.49	89.40	88.70	89.23	74.00	17.00	63.09
	MPW	2.76e-1	6.99e-2	1.08e-1	1.64e-1	1.66e-1	2.02e-1	1.28e-2	3.57e-7	1.88e-3
2D Sphere	ACC	**93.28**	64.33	74.86	85.15	85.33	82.60	66.32	18.04	4.78
	MPW	8.29e-1	1.99e-2	1.62e-2	6.18e-2	3.17e-2	4.08e-2	3.82e-4	1.36e-5	2.60e-6
3D Sphere	ACC	**92.99**	74.48	81.47	86.04	85.62	85.83	66.39	14.00	5.01
	MPW	8.27e-1	3.04e-2	9.78e-3	8.44e-2	1.94e-2	2.71e-2	2.00e-3	6.64e-5	1.11e-5
5D Sphere	ACC	**93.22**	82.40	87.15	88.99	91.45	91.88	74.53	26.09	10.39
	MPW	7.65e-1	5.27e-2	8.68e-3	1.36e-1	1.60e-2	2.12e-2	4.60e-4	8.85e-5	2.10e-5

Table 14: Accuracy of various rules predicting the RCV winner along with normalized multiplicative weights for 90% sample sizes.

		RCV	Plurality	Borda	Harm.	Cope.	MM	Bucklin	Pl. Veto	Veto
M, $\phi = 0.001$	ACC	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	25.73
	MPW	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	5.12e-8
M, $\phi = 0.01$	ACC	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	30.07
	MPW	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.11e-7
M, $\phi = 0.05$	ACC	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	36.37
	MPW	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	3.67e-7
M, $\phi = 0.1$	ACC	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	46.31
	MPW	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	3.10e-6
M, $\phi = 0.25$	ACC	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	71.43
	MPW	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	1.25e-1	4.28e-4
M, $\phi = 0.5$	ACC	100.00	100.00	100.00	100.00	100.00	100.00	81.80	100.00	88.15
	MPW	1.41e-1	1.41e-1	1.41e-1	1.41e-1	1.41e-1	1.41e-1	3.01e-3	1.41e-1	1.33e-2
M, $\phi = 0.75$	ACC	**97.93**	87.58	91.00	94.00	97.19	97.09	79.72	88.00	73.32
	MPW	2.50e-1	2.19e-2	1.12e-1	6.55e-2	2.54e-1	2.35e-1	3.93e-3	5.48e-2	2.48e-3
M, $\phi = 0.95$	ACC	**92.56**	60.37	78.52	74.00	83.37	85.41	33.82	43.00	46.05
	MPW	7.15e-1	9.70e-4	3.56e-2	1.64e-2	9.68e-2	1.35e-1	2.56e-5	1.71e-4	3.87e-4
M, $\phi = 0.99$	ACC	**91.59**	56.60	74.51	67.00	77.41	78.90	23.50	19.00	40.30
	MPW	8.92e-1	5.89e-3	1.27e-2	1.01e-2	2.37e-2	5.52e-2	1.33e-5	4.48e-6	8.72e-5
M, $\phi = 0.999$	ACC	**91.87**	61.81	71.54	72.00	74.71	78.77	24.67	18.00	45.35
	MPW	7.43e-1	1.73e-3	8.08e-2	2.11e-2	8.69e-2	6.57e-2	3.23e-5	2.79e-6	2.91e-4
IC	ACC	**87.32**	56.19	65.05	63.00	76.22	77.65	27.77	18.00	47.65
	MPW	5.16e-1	5.90e-3	9.37e-2	3.84e-2	7.15e-2	2.74e-1	1.94e-5	1.48e-5	8.12e-4
Urn, $\alpha = 0.01$	ACC	**94.33**	57.30	64.56	66.00	71.34	74.04	34.68	21.00	37.07
	MPW	7.88e-1	2.98e-3	3.38e-2	2.52e-2	5.01e-2	9.99e-2	1.33e-4	3.65e-6	6.09e-5
Urn, $\alpha = 0.02$	ACC	**92.60**	54.52	70.07	71.00	76.63	75.04	42.45	17.00	43.59
	MPW	8.95e-1	3.22e-3	1.07e-2	1.54e-2	3.33e-2	4.13e-2	5.21e-4	6.74e-6	2.12e-4
Urn, $\alpha = 0.05$	ACC	**94.93**	74.04	72.49	80.00	82.28	83.40	48.22	16.00	32.43
	MPW	7.29e-1	6.22e-3	1.77e-2	3.97e-2	9.96e-2	1.08e-1	3.20e-4	9.07e-7	1.44e-5
Urn, $\alpha = 0.1$	ACC	**96.32**	73.05	69.00	78.00	79.68	83.42	57.50	16.00	31.45
	MPW	7.75e-1	1.43e-2	9.22e-3	7.24e-2	6.91e-2	5.92e-2	1.12e-3	3.19e-6	9.28e-6
Urn, $\alpha = 0.2$	ACC	**99.00**	70.56	72.56	81.00	87.99	87.53	66.46	19.97	32.07
	MPW	6.64e-1	8.47e-2	7.55e-3	1.19e-1	4.22e-2	8.04e-2	1.43e-3	1.44e-6	5.42e-6
Urn, $\alpha = 0.5$	ACC	**99.45**	87.54	74.00	85.00	86.33	87.46	79.17	39.00	29.78
	MPW	4.54e-1	1.04e-1	2.11e-2	1.58e-1	1.09e-1	1.37e-1	1.65e-2	1.58e-6	1.66e-6
SC	ACC	**98.05**	84.49	77.41	91.00	87.05	86.93	68.83	15.00	40.70
	MPW	6.01e-1	3.33e-2	7.44e-3	2.06e-1	7.37e-2	7.37e-2	4.84e-3	1.90e-7	1.24e-5
Conitzer SP	ACC	**92.24**	37.14	35.00	62.00	36.49	36.51	53.90	0.00	33.71
	MPW	9.86e-1	1.75e-3	3.07e-5	1.14e-2	2.93e-5	2.98e-5	1.09e-3	2.88e-7	3.24e-5
Conitzer SPOC	ACC	**91.34**	61.62	63.12	69.00	65.14	63.41	23.17	22.00	39.53
	MPW	8.62e-1	2.10e-2	1.18e-2	8.76e-2	8.30e-3	9.35e-3	1.82e-5	1.66e-5	1.20e-4
Walsh SP	ACC	**98.56**	86.45	88.00	87.00	87.00	88.00	53.52	0.00	33.42
	MPW	3.28e-1	1.41e-1	1.27e-1	1.49e-1	1.27e-1	1.27e-1	9.48e-5	1.27e-8	2.68e-6
1D Interval	ACC	**95.71**	63.06	43.00	79.00	53.00	51.00	49.73	23.00	20.86
	MPW	9.86e-1	3.01e-3	3.16e-5	9.79e-3	3.29e-4	3.27e-4	1.33e-4	2.42e-6	2.70e-6
2D Square	ACC	**97.09**	65.26	73.00	79.00	79.87	78.67	64.29	23.00	37.40
	MPW	8.82e-1	1.06e-3	4.03e-3	3.77e-2	3.64e-2	3.61e-2	2.77e-3	1.52e-6	2.92e-5
3D Cube	ACC	**96.05**	61.54	78.00	85.00	85.47	86.85	68.16	21.00	56.39
	MPW	5.35e-1	7.81e-3	2.80e-2	1.12e-1	1.51e-1	1.59e-1	6.05e-3	2.82e-7	5.16e-4
5D Cube	ACC	**96.42**	78.85	88.67	88.00	91.09	91.38	71.91	22.00	64.40
	MPW	3.22e-1	5.53e-3	6.57e-2	9.63e-2	2.33e-1	2.69e-1	7.02e-3	4.42e-7	1.43e-3
10D Cube	ACC	**96.94**	75.98	85.44	92.00	94.58	96.47	75.10	24.00	69.52
	MPW	3.34e-1	2.46e-2	9.19e-2	9.37e-2	2.16e-1	2.34e-1	3.94e-3	4.44e-8	2.22e-3
20D Cube	ACC	**97.03**	84.59	86.00	92.37	92.55	93.63	76.69	17.00	63.66
	MPW	3.51e-1	5.28e-2	7.43e-2	1.03e-1	1.88e-1	2.21e-1	9.03e-3	1.90e-7	8.98e-4
2D Sphere	ACC	**97.22**	64.59	77.38	89.00	86.47	85.98	67.41	16.00	4.48
	MPW	9.68e-1	2.43e-3	2.21e-3	6.63e-3	1.02e-2	1.02e-2	1.79e-5	9.31e-7	1.01e-7
3D Sphere	ACC	**99.11**	76.45	83.00	87.00	88.35	88.04	67.06	15.00	3.87
	MPW	9.85e-1	1.34e-3	7.37e-4	7.48e-3	2.06e-3	2.96e-3	4.46e-5	1.87e-6	2.02e-7
5D Sphere	ACC	**97.55**	81.99	87.00	89.00	93.81	91.50	74.63	24.00	8.89
	MPW	9.66e-1	5.61e-3	6.63e-4	2.14e-2	2.90e-3	3.29e-3	1.50e-5	3.68e-6	6.45e-7

Table 15: Accuracy of various rules predicting the RCV winner along with normalized multiplicative weights for 100% sample sizes.

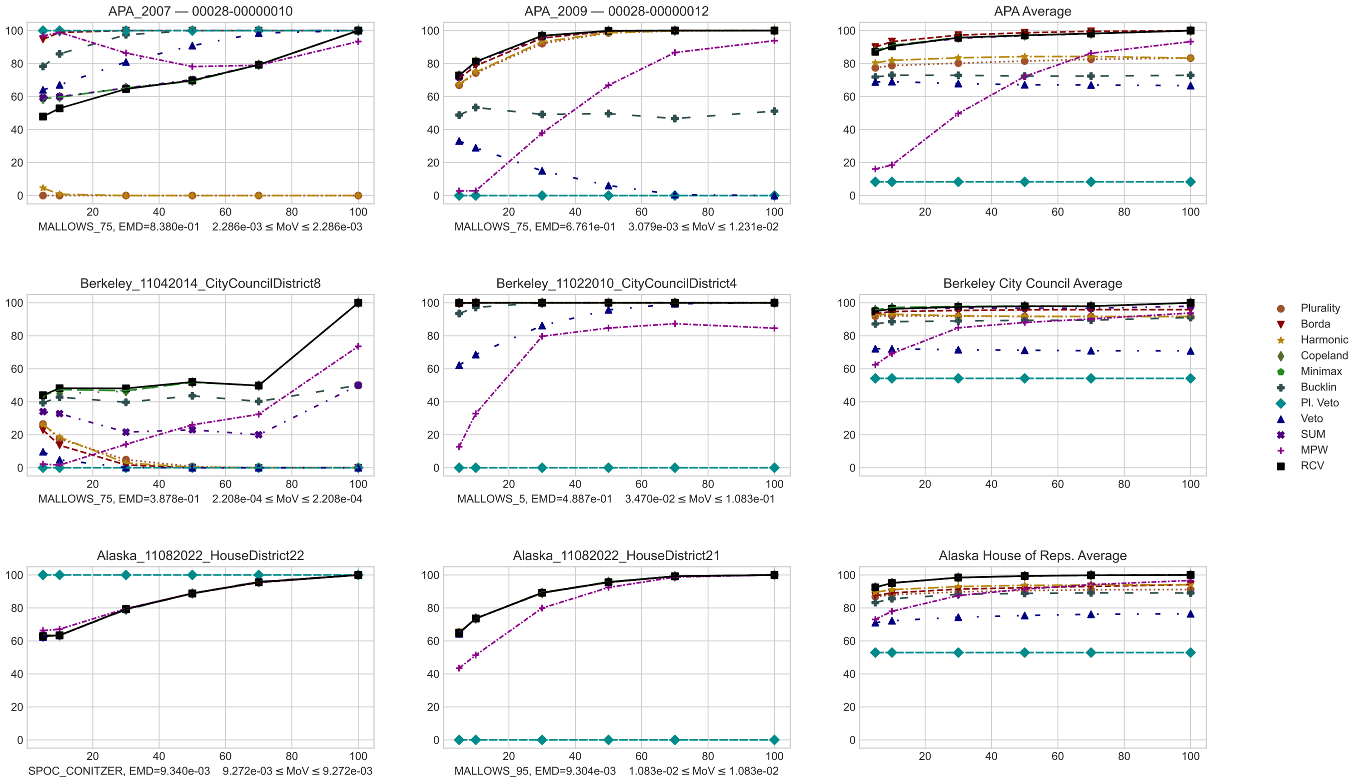

Figure 3: Summary and individual plots for the APA, Berkeley City Council, and Alaska House of Representatives datasets. We show the closest statistical culture and bounds on the MoV for individual elections. EMD is the positionwise distance [Szufa *et al.*, 2020].

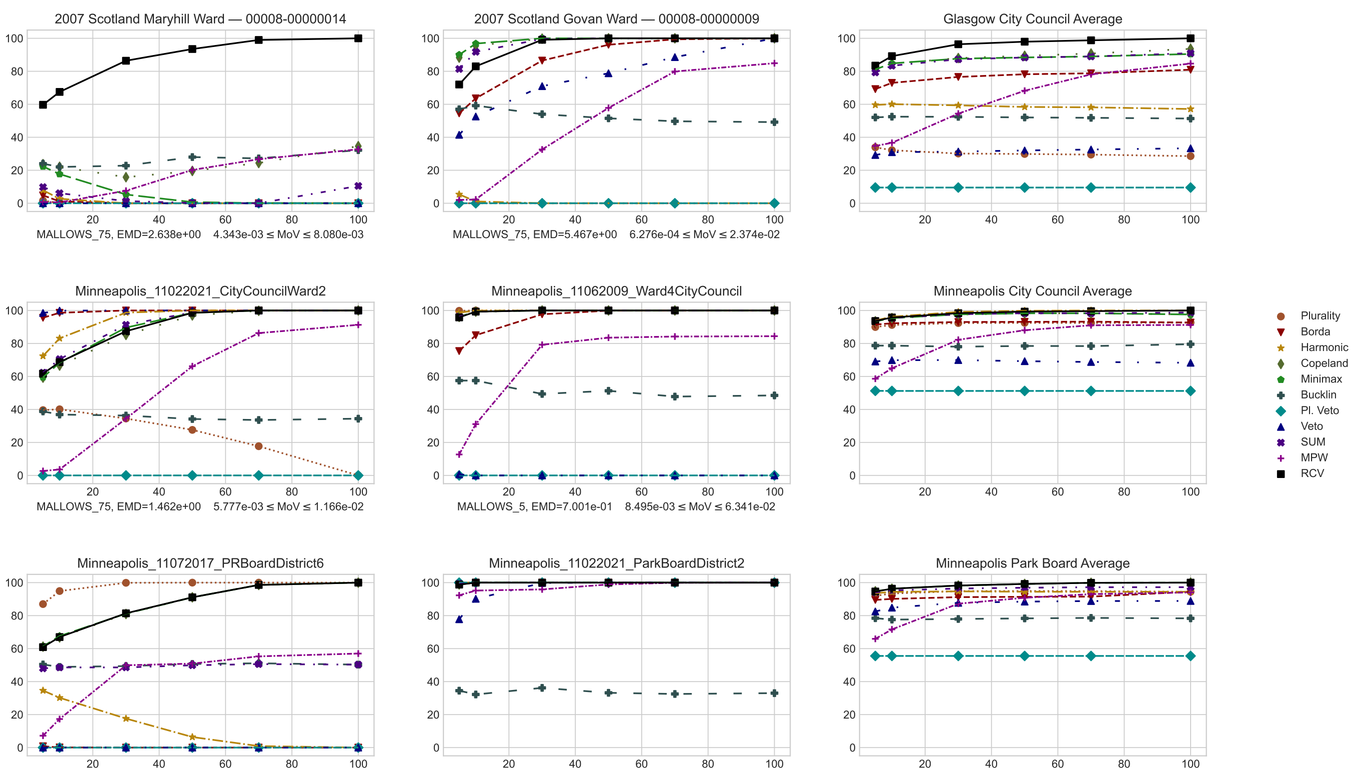

Figure 4: Summary and individual plots for the Glasgow City Council, Minneapolis City Council, and Minneapolis Park Board datasets. We show the closest statistical culture and bounds on the MoV for individual elections. EMD is the positionwise distance [Szufa *et al.*, 2020].

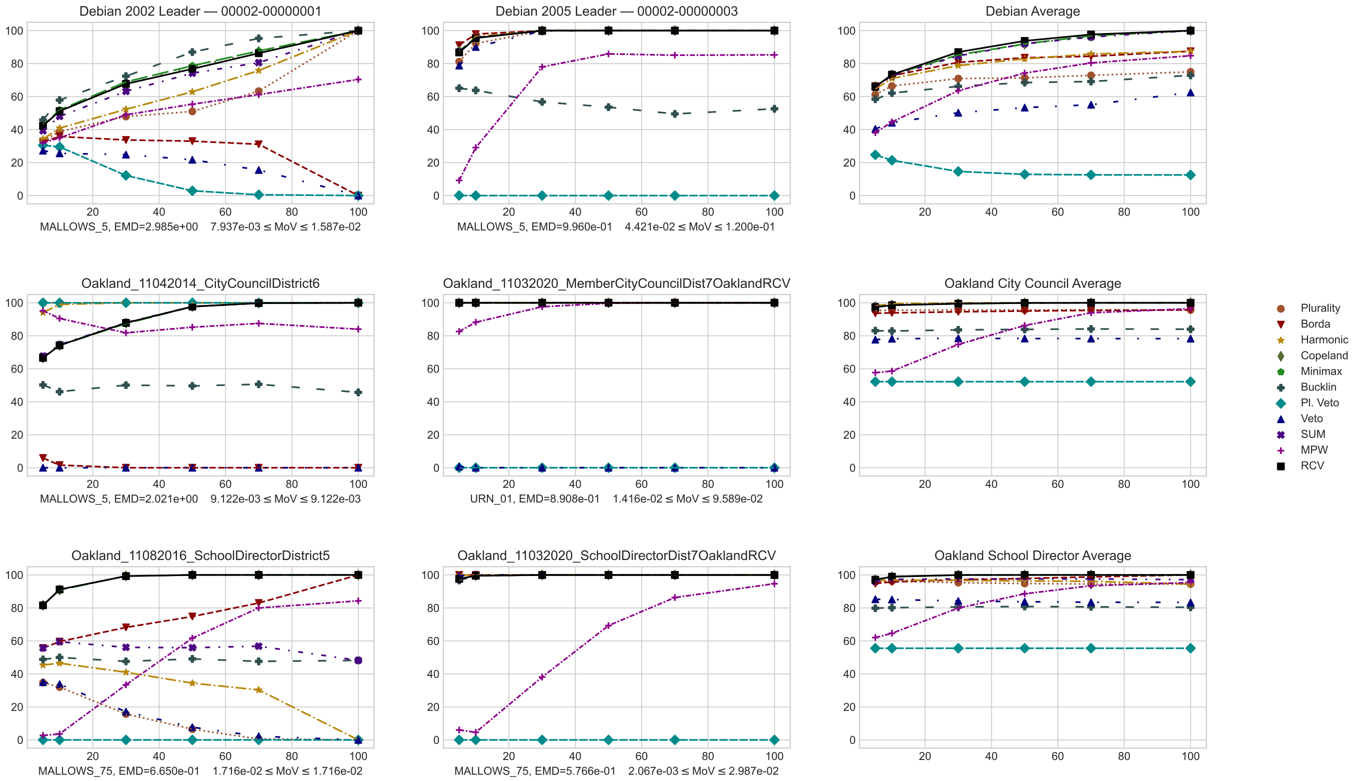

Figure 5: Summary and individual plots for the Debian, Oakland City Council, and Oakland School Director. We show the closest statistical culture and bounds on the MoV for individual elections. EMD is the positionwise distance [Szufa *et al.*, 2020].

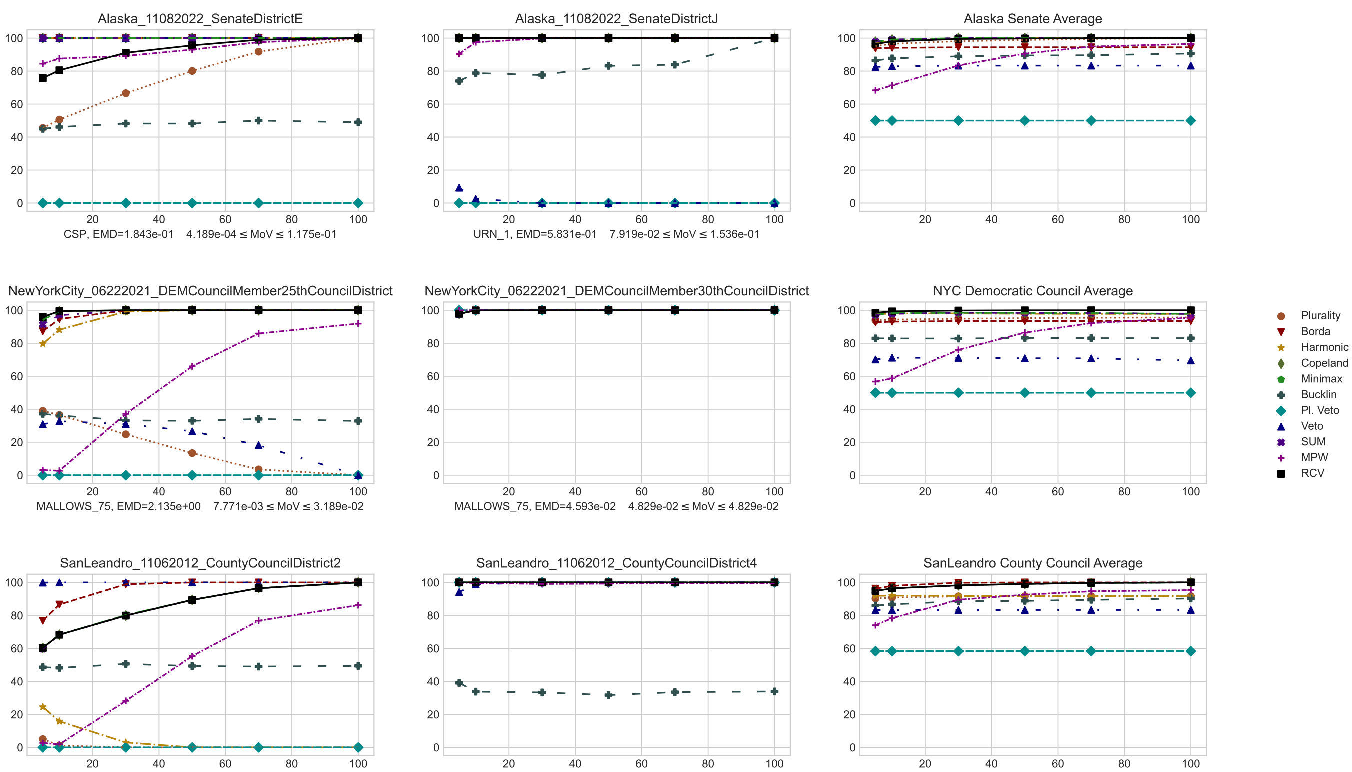

Figure 6: Summary and individual plots for the Alaska Senate, Ney York City Democratic Council, and San Leandro County Council datasets. We show the closest statistical culture and bounds on the MoV for individual elections. EMD is the positionwise distance [Szufa *et al.*, 2020].